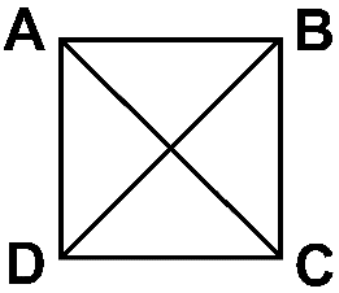
Two Different Lengths

Puzzle 1



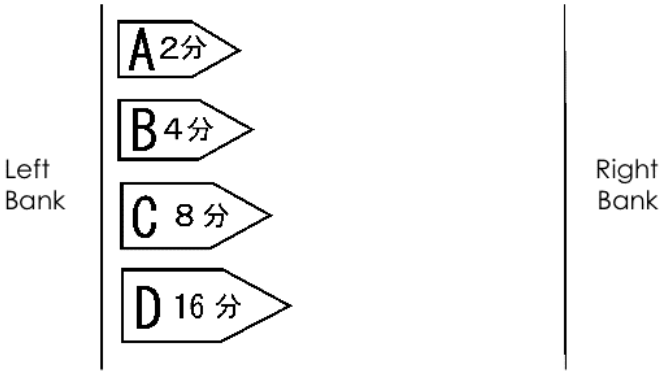
Let's look at a square of side-length 1. There are six straight lines that connect any two vertices of the square. The length of these lines is either

AB = BC = CD = DA = 1
or
AC = BD= √2.

Can you rearrange the four vertices so that they are on a plane and there are still only two different lengths connecting any two points? How many different arrangements can you think of? No two points are allowed to be in the same location.

(Puzzle by Dick Hess)

Puzzle 2 Crossing the River



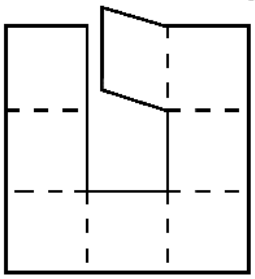
Four boats A–D are on the left bank of a river. To cross the river to the right bank, Boat A takes 2 minutes; Boat B takes 4 minutes; Boat C takes 8 minutes; and Boat D takes 16 minutes.

The four boats need to go to the right bank, but there is only one boatman. One boat can pull another boat, but it will take the same amount of time as the slower of the two boats to cross the river.

If the boatman uses one boat to pull another boat to the right bank and returns to the left bank in one boat, how many minutes will it take for him to move all the boats to the right bank? Find the shortest time (ignore the time he spends changing and connecting boats).

Puzzle 3

Making a Die



Fold a square into a 3 x 3 grid. Cut out the square in the middle and cut one section as shown here. You now have eight small squares. Can you fold this paper and make it into a cube? Since a cube only has six surfaces, you need to overlap some squares.

(Puzzle by Edward Hordern)

Puzzle 4

What Comes Next?

What comes after 66?

2, 4, 6, 30, 32, 34, 36, 40, 42, 44, 46, 50, 52, 54, 56, 60, 62, 64, 66, ?

Puzzle 5 Diamond Puzzle

A square is missing one-quarter of its area; we want to turn it into the diamond shape in Figure 1 by cutting it into two pieces and rearranging them.

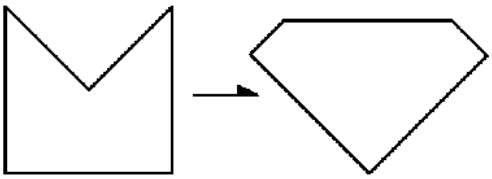


Figure 1.

In Figure 2, we cut the square into four pieces and rearranged them. Can you find a way to cut and rearrange fewer pieces and make a diamond?

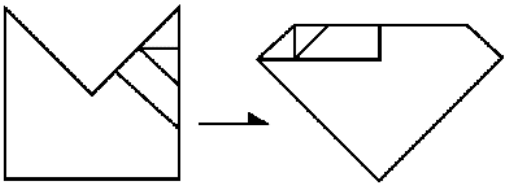
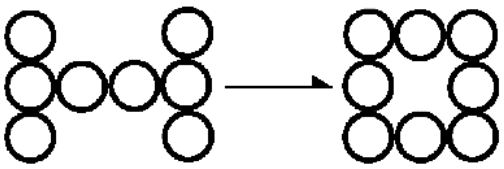


Figure 2.

The Water Puzzle

Puzzle 6



Eight coins are arranged to look like the letter H. By sliding one coin around at a time, we want to arrange the eight coins to look like the letter O. However, there is a strict rule: A moving coin needs to stop in a position where it touches at least two coins.

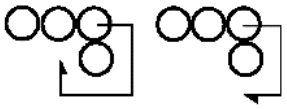


Figure 1. Figure 2.

In Figure 1, the moving coin ends up touching two coins, but in the Figure 2, the moving coin ends up touching only one coin, so it is prohibited.

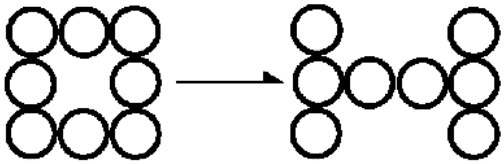
Figure 3.

In Figure 3, a moving coin is indeed touching two coins, but this coin will not stop right there—it will slide on; therefore, such a move is also prohibited.

Please get eight real coins and play with them. If you form the letter O in five moves, I will be really impressed.

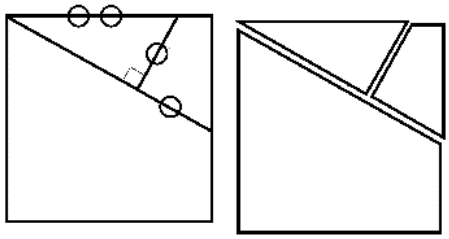
By the way, I call this puzzle "The Water Puzzle" because it changes H to O (H_2O).

Puzzle 7 The Retaw Puzzle



Continue from Puzzle 6. This time, each coin needs to be moved around so that the eight coins arranged as the letter O change into the letter H. The minimum number of moves is seven—this puzzle is much more difficult.

Puzzle 8 Three Times More



I divided a square into three parts. The lines labeled with one circle are the same length, and the length of the line with two circles is twice as long as the lines with one circle.

Get some paper, cut three squares, and then cut each of them into three pieces as shown here.

Now, using the nine pieces cut from the squares, create a square whose area is three times that of the original square.

This puzzle will be less difficult if you actually make the pieces.

Triangular Arithmetic

Puzzle 9

Arrange the numbers 1 to n (where n is a "triangular" number) to form an equilateral triangle in such a way that the difference between two neighboring numbers is right below them. Figure 1 shows some samples:

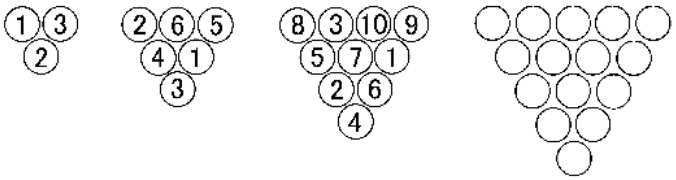
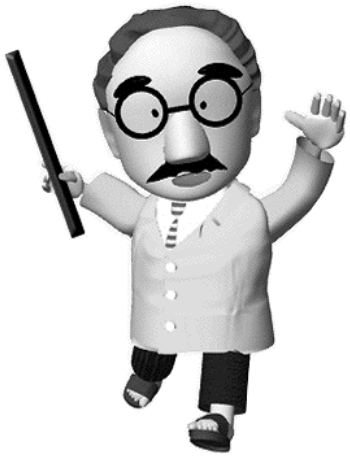
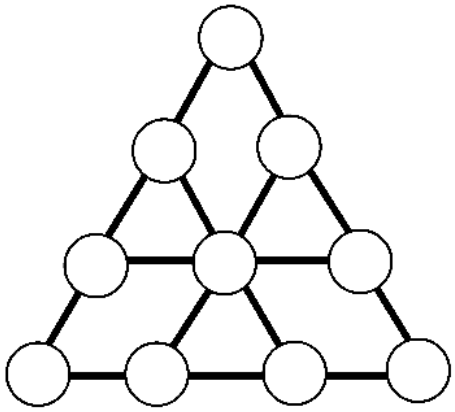


Figure 1.

Now, arrange the numbers 1to15 to create such a triangle.



Puzzle 10 Triangular Solitaire



Connect ten circles in the form of an equilateral triangle as shown. Put one coin on each circle and follow these steps:

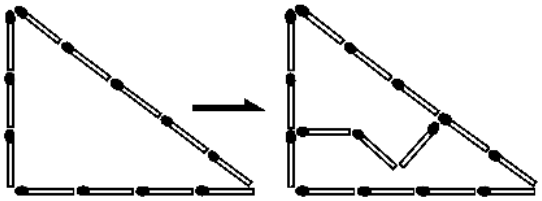
1. Remove one of the coins.
2. Pick another coin. If there is a coin on a circle adjacent to the coin you chose and the circle beyond that coin is empty, then you can jump over that coin and take it away.
3. Repeat Step 2. You are done when there is only one coin left.

Advanced: Try to solve this in as few steps as possible. Jumping continuously is counted as one step.

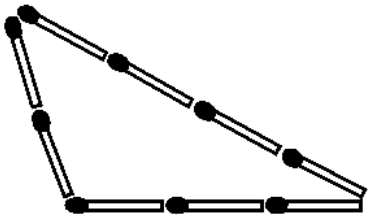
The Same Areas

Puzzle 11

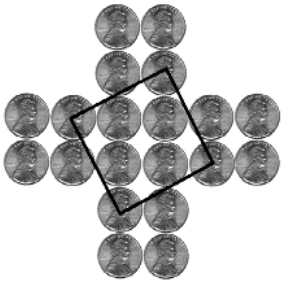
Using three matchsticks, bisect the area of a triangle that is made up of three, four, and five matchsticks on each side, respectively. This can be solved as shown in Figure 1.



Now, decrease the number of matchsticks by one on each side and try to bisect the area of the triangle using only two matchsticks.



Puzzle 12 Lose Those Squares



Arrange 20 coins as in the picture. We can create 21 squares by connecting the centers of four of the coins (an example is shown). Now remove as few coins as possible until no square can be made.

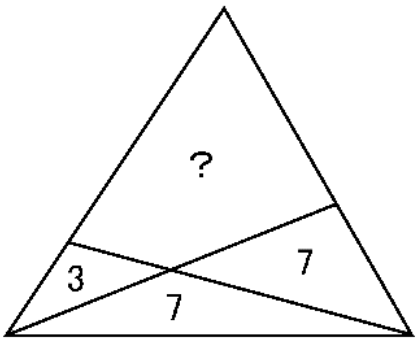
Puzzle 13 Two Choice Question

生徒＼問題	1	2	3	4	5	6	7	8	9	10	得点
A	○	×	×	○	×	×	○	○	×	○	80
B	×	○	×	×	×	○	×	○	×	×	20
C	○	×	○	○	○	×	○	○	○	○	70
D	×	×	○	×	○	×	×	×	○	×	?

There are ten questions for which you can choose O or X as the answer. Each right answer is worth ten points. The results for Students A, B, and C are as shown in the table. But the teacher has forgotten to write down the total score for Student D, and he also has lost the correct answer for each question. From the table, find out D's score.

Puzzle 14

Triangulation



A triangle is divided into four parts by two straight lines as shown. The area ratio of the three parts is 3:7:7. What is the size of the fourth area?

Puzzle 15

Rock, Paper, Scissors

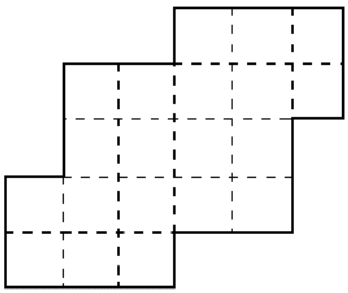
A boy and a girl played Rock, Paper, Scissors ten times. The boy used three rocks, six scissors, and one paper. The girl used two rocks, four scissors, and four papers.

There was never a tie, and the order in which the boy and girl used rocks, papers, and scissors is unknown. Who has won by how many wins?

(Puzzle by Yoshinao Katagiri)

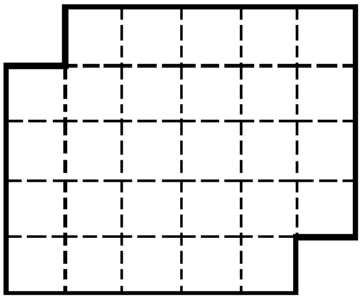
Puzzle 16 Four Solutions of Four Pieces I

Cut this shape into four identical pieces. Don't stop after you find one solution—I've found four.



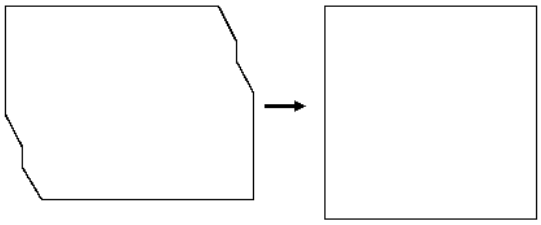
Puzzle 17 Four Solutions of Four Pieces II

Use the same rule as in Puzzle16; try to find four solutions again.



Puzzle 18

Square



Cut the shape on the left into two parts and reconnect them to make a square.

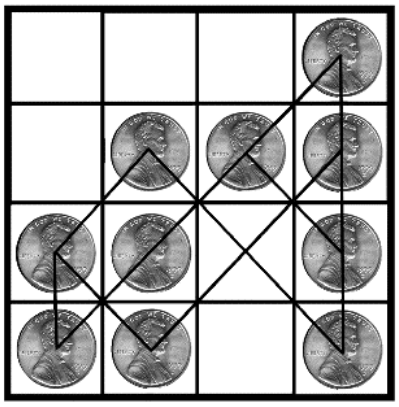
Puzzle 19

Simultaneous Equations

$$\square\square \times \square = \square\square$$
$$\square \times \square = \square\square$$

Fill in each of the squares with one each of nine digits 1 to 9 so that both equations are correct.

Puzzle 20 Even Number Sequence



Ten coins are arranged on a 4 x 4 board, in the picture. As shown here, there are eight (horizontal, vertical, and diagonal) sequences where an even number of coins is lined up. Rearrange the ten coins so that you have the largest number of sequences with an even number of coins. Try to find an arrangement where you have the least number of sequences as well.

Puzzle 21

Age Guessing

Father: I have just realized that, if I switch the number in the ones digit and in the tens digit of my age, I get your age.
Son: Tomorrow you will be exactly twice as old as I.

How old are they as of today? Don't be too quick to answer.

Puzzle 22

Divide by Four

This shape is half of a regular hexagon. Divide it into four identical shapes; you can flip over shapes. I've found two solutions.

Puzzle 23 Can You?

Can you move the "minus" to make this equal to nine fifty?

(Puzzle by Mel Stover)

Puzzle 24 To Have or Not to Have?

LAUGHING has it, but not CRYING.
HIJACK has it, but not TERRORISM.
FIRST has it, but not SECOND.
AFGHANISTAN has it, but not TAJIKISTAN.
CALMNESS has it, but not NOISE.
DEFINE has it, but not DECIDE.

What is it?

Pentominos

Puzzle 25

Five squares can be connected in 12 different ways, as shown in Figure 1. We consider two shapes to be the same if they become identical by flipping one of them over.

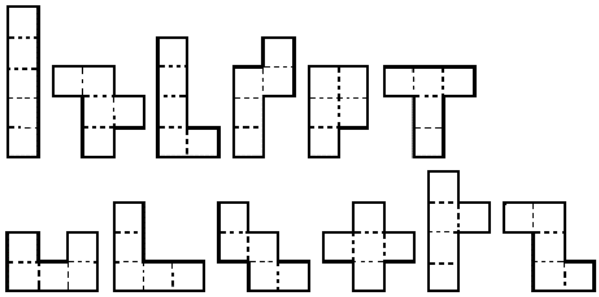


Figure 1.

Of these 12 shapes, find two different shapes that can fill the following shape in Figure 2. How many different solutions can you find? You are allowed to flip over the shapes.

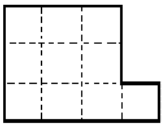


Figure 2.

Puzzle 26

Pandigital
(KOMACHI) Fraction I

$$\frac{\square}{\square\square} + \frac{\square}{\square\square} + \frac{\square}{\square\square} = 1$$

Fill out the squares in the equation using the numbers 1 to 9 once and only once. Two boxes together is a two-digit number.

Puzzle 27

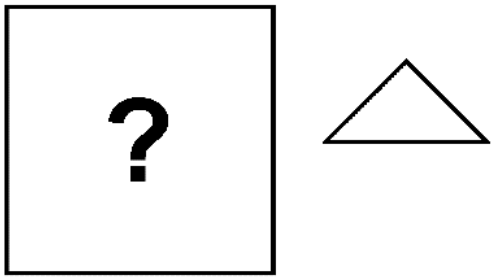
Pandigital
(KOMACHI) Fraction II

$$\frac{\square}{\square\times\square} + \frac{\square}{\square\times\square} + \frac{\square}{\square\times\square} = 1$$

Fill out the squares in the equation using the numbers 1 to 9 exactly once.

Puzzle 28

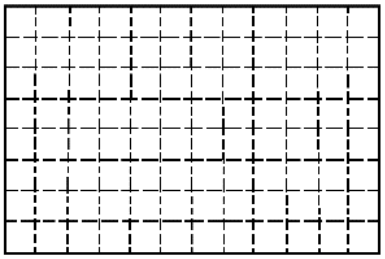
Triangulate a Square



Try to fill up this square using only right isosceles triangles (as shown). Each triangle should be a different size—try to use as few triangles as possible. So far, I know of two solutions.

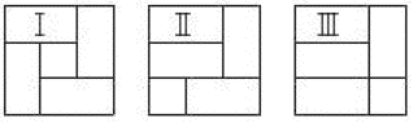
Puzzle 29

Challenge
by Don Knuth



Fill an 8 x 12 grid using 32 1 x 3 pieces so that a "cross" is not formed. A "cross" is formed when two lines intersect at a four-way junction.

Note: Japanese tatami mats are arranged as in Examples I and II such that no line crosses the room. Tatami mats would never be arranged as in Example III. Don Knuth created this puzzle when he visited Japan and had a conversation about tatami mats with me. Since the given condition is severe, a solution can be found easily. If there are no restrictions, there are 51,493 different solutions. Thanks to this restriction, there are only two solutions.



Square Numbers

Puzzle 30

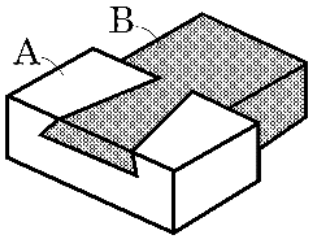
Arrange the numbers 1–15 so that the sum of two neighboring numbers is always a square number.

Hint: You can find a solution using pencil and paper. The same square number will be used more than once.

(With help from Goro Tanaka)

Interlocking Pieces?

Puzzle 31



A white wooden piece and a black wooden piece are interlocked as shown.

Can you guess how they might come apart easily? There is no hollow space inside. The bottom view is just a black rectangle abutting a white rectangle.