The Binomial distribution

A random variable x has a binomial distribution if its probability distribution is given by:

$$f(x) = \binom{n}{x} p^x (1-p)^{(n-x)}$$

where

$$\begin{aligned} f(x) = &\quad \text{probability of } x \text{ successes in } n \text{ trials} \\ \tbinom{n}{x} = \frac{n!}{x!(n-x)!} = &\quad \text{the number of possible outcomes resulting in } x \text{ successes in } n \text{ trials} \\ p = &\quad \text{the probability of success in any trial} \\ (1-p) = &\quad \text{the probability of failure in any trial} \end{aligned}$$

Illustration

Suppose the Nice Clothing Store would like to estimate the probability that out of the next 4 customers (trials), 3 make a purchase (success), when the manager knows that the probability of a purchase by a single customer is 0.20.

Basically the problem consists of finding the probability of 3 successes out of 4 trials, that is:

$$\begin{aligned} f(3) &= \binom{4}{3} p^3 (1-p)^{(4-3)} \\ &= 4p^3(1-p) \\ &= 4(0.20^3)(0.80) \\ &= 4(0.0064) \\ &= 0.0256 \end{aligned}$$

given that the number of possible outcomes with 3 successes in 4 trials is:

$$\binom{n}{x} = \frac{n!}{x!(n-x)!} = \frac{4!}{3!(4-3)!} = \frac{4!}{3!1!} = 4$$

Reference: Anderson, D.R., D.J. Sweeny, and T.A. Williams (1999): *Statistics for Business and Economics.* South–Western.