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Math Fluency: Accuracy Versus Speed in Preoperational and Concrete Operational First and Second Grade Children

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Abstract Cognitive abilities as well as math fluency play an important role in mathematical skills. Understanding the relationship between cognitive abilities and mathematical skills is imperative to teaching effective arithmetic skills. The present study aimed to investigate the relationship between cognitive ability and math fluency with 38 first and second grade elementary aged children. Results demonstrate that preoperational children lacked the speed of concrete operational children but achieved comparable levels of accuracy when completing arithmetic problems.

Keywords Child development · Arithmetic · Math fluency · Academic achievement · Childhood education

Competence in arithmetic is an important goal of early schooling, and research on the early development of arithmetic-related skills is important for understanding and optimizing the transitions children undergo as academic knowledge is acquired (Klein and Bisanz 2000). The acquisition and application of arithmetic skills, such as counting and simple addition and subtraction, hold great societal importance due to the demands of formal schooling, daily activities, and employment (Mullis et al. 2001; Rivera-Batiz 1992; Rourke and Conway 1997). Research has found that mathematical difficulties do not exist only for individuals with low cognitive abilities (Siegel 1988). Some students have difficulty with mathematics because they have not become fluent in basic mathematics computation skills (Binder 1996; Hasselbring et al. 1988). Students who possess mathematical fluency (accuracy and speed) usually maintain those skills longer, are able to stay on-task for extended periods of time, and are better able to resist distractions (Binder 1996; Hasselbring et al. 1988; Lindsley 1996). According to Binder (1996), students who can respond fluently with basic mathematic skills are more successful in applying those skills to new mathematics tasks. Binder (1990) determined that when a combination of accuracy and speed of performance optimizes a specific behavioral outcome, the behavior that led to the performance of that task could be considered "mastered".

While fluency plays a major role in problem solving, cognition allows children to acquire, retain, and use arithmetic skills.

Fluency is the fluid combination of accuracy plus speed of response that characterizes competent behavior (Binder 1996; Haughton 1980). Individuals high in fluency can perform tasks quickly and accurately, thus retaining more resources that can be used for comprehension (Therrien 2004). An individual who lacks fluency or has fluency problems spends a great deal of their cognitive resources (i.e., attention, working memory) on decoding and has limited resources left for comprehension and learning more complex tasks (Dahaene 1997). Cognitive processing theories indicate that individuals have a limited cognitive capacity which makes attending to multiple tasks simultaneously challenging unless some of the tasks require less time, cognitive effort, working memory, and/or attention (Delazer et al. 2003; Pellegrino and Goldman 1987). Researchers have demonstrated that with daily practice of arithmetic problems designed to improve accuracy and speed (fluency), students are able to achieve higher performance frequencies and progress productively in their math curriculum (Haughton 1972; Starlin 1972). Additionally,

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researchers have found that students who are fluent in math skills show lower levels of math anxiety (Cates and Rhymer 2003) and will engage in math activities more often than students that lack fluency (Billington and Skinner 2002).

Fluency effects have been examined in several demonstration studies; however, rarely in experimental investigations (Singer-Dudek and Greer 2005). Fewer studies have looked specifically at the development of math fluency. Those studies found correlations between fast rate instruction and each of the benefits associated with fluency theory (Singer-Dudek and Greer 2005; Binder 1996; Ivarie 1986). However, those studies focused on reading related competencies, with relatively little regard to the development and maintenance of mathematical skills, but differences do exist in children's math abilities at a young age (Floyd et al. 2003). Although the literature on math fluency is sparse, the importance of the concept is evident. The National Council of Teachers Mathematics (NCTM 2000) listed fluent computation as a goal for mathematics instruction, and failure to rapidly recall basic facts as a characteristic often associated with mathematical disabilities. In 2006 NCTM, produced a new document titled, "Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics", which was developed as a framework for curriculum developers that identified important content for each grade level that could build connected and integrated mathematical understanding. The document highlighted "developing quick recall of basic addition facts and related subtraction facts and fluency with multidigit addition and subtraction", as a curriculum focal point for Grade 2. This document lends support to the importance of understanding the development of fluency at an early age.

Piaget's cognitive developmental theory aids in understanding the development of children's cognitive abilities. His stages of cognitive development explain how cognitive skills are acquired and progress over time. These stages are: sensory motor, preoperational, concrete operational and formal operational. Two stages that occur during the school-age are preoperational and concrete operational (Wadsworth 1996). According to Piaget and Inhelder (1969), preoperational children think intuitively and conceptually, but not logically. The concrete operational stage is characterized by the ability to reason logically despite changes in dimensions of objects (Martini 2004). Conservation and reversibility are necessary for mathematical success and are developed in children who have obtained concrete operations. Although Piaget identified reversibility and class inclusion as prerequisites for computing addition, subtraction, and negative numbers, many first-graders or 6-year olds are "taught" addition before they can understand it. This means that although a first-grader can answer a math problem it is most likely that he/she is utilizing rote-memorization to identify the answer and does not actually understand the process involved in the computation.

Although the literature provides some information regarding the role of cognition and fluency on mathematical performance relatively little is known about the relationship between them. Studies have shown that cognition enables the child to process, transform, reduce, elaborate, store, and recover information needed to perform arithmetic problems while fluency enables the child to complete the arithmetic problems efficiently and effectively. An extensive literature review revealed that few studies have examined the development of fluency in young children and instead focus on the effects of fluency. In addition, no studies have focused on the direct relationship between cognition and fluency.

The current study investigated whether there are measurable differences in fluency among concrete and preoperational first and second grade children, and examined the relationship between accuracy and speed based on the participant's cognitive developmental level. The hypotheses were: (1) Concrete operational children and preoperational children will differ significantly on math fluency (as measured by speed and accuracy of performing simple math calculations), (2) Concrete operational children and preoperational children will differ significantly on speed (number answered) of performing simple math calculations, and (3) There will be no significant difference between concrete operational and preoperational children's math performance rate of accuracy (as measured by percentage correct simple math calculations).

Methods

Participants

This study was conducted as part of a larger longitudinal study examining the effects of metacognitive instruction on young school age children. The study consisted of 39 first and second grade children from an elementary school in Oak Park, Illinois. The total sample included 22 (56%) females and 17 (44%) males. There were 17 (44%) first graders and 22 (56%) second graders, with the total sample consisting of 59% Caucasian children, 39% African-American children, and 3% were children of other ethnicities.

Materials

Mathematical fluency ability was assessed using the Woodcock Johnson Tests of Achievement, 3rd Edition (WJ-III; Woodcock et al. 2001). The Math Fluency subtest on the WJ-III requires the child to complete as many simple one-digit addition, subtraction, and multiplication problems as quickly as possible within a 3-min time limit. This subtest has no basal or ceiling rules with 160 total number of problems. This timed task determines the accuracy, speed, and fluency of basic rote mathematical calculations by the total number of correct and incorrect items each child completes in 3 min. Accuracy for each participant was determined by calculating percentage correct (how many answered/how many correct). Speed was determined by the number of actual problems answered. (i.e., child completed 10 problems, score for speed = 10). Fluency was then determined by combining the accuracy and speed score.

The cognitive developmental level of each participant was determined by their performance on two conservation tasks, Conservation of Number and Conservation of Substance. These tasks are based on Piaget's theory of cognitive development, which asserts that a child's developing ability to conceptualize conservation is related to his/her level of cognitive development (Wadsworth 1996). Conservation is the ability to conceptualize whether the amount of matter remains the same or changes among the trials presented for each conservation task (Wadsworth 1996). Specifically, these conservation tasks evaluate whether a child can conserve one aspect of quantity while another aspect changes.

The Conservation of Number task involves three trials using six black checkers and six red checkers. The first trial consisted of placing the black checkers in a row, equal distance to each other and placing the red checkers in a similar equidistant row parallel to the black checkers. The child is then asked: "Are there as many red checkers as black checkers or is there more of one kind?" If the child does not respond correctly, they are prompted to count each row until they conclude that there are as many red and black checkers. The purpose of this trial is to ensure that the child understands that this task begins with an equal number of red and black checkers. The second trial involves elongating the row of red checkers while the row of black checkers remains the same. Again, the child is asked: "Are there as many red checkers as black checkers, or is there more of one kind?" The child is given 1 point when a correct response is provided. The child is then asked, "How did you know?" All responses are recorded verbatim and the child is given another point if a correct response is provided. Next, the red checkers are arranged into a circle while the black checkers remain in a line. The child is asked the same questions and all responses are scored and recorded verbatim.

The Conservation of Substance task involved two balls of equal amounts of play-doh®. On the first trial, the child is asked, "Is there as much play-doh in both shapes, or is there more in one than the other?" If the child answers there is more, than the child is asked "to fix it" until the child believes that there is equal amounts of play-doh in both shapes. For the second trial, the examiner will roll one of the balls into a tubular shape, and place it vertically alongside the ball shaped play-doh used in the first trial. The child is then asked, "Is there as much play-doh in both shapes or is there more in one than the other?" The child will receive 1 point if the correct response is given. The child will then be asked how he/she knows, and his/her response will be recorded verbatim. The child will again receive a point if the correct response is provided.

A total score ranging from 0 to 6 based on the Conservation of Number and Conservation of Substance tasks was calculated for each child. An overall score of 0 or 1 indicates that the child failed both conservation tasks, thus determining that the child is functioning at the preoperational stage of cognitive development. A score of 5 or 6 indicates that a child successfully completed both tasks, and is functioning at the concrete operational stage of cognitive development. Scores ranging from 2 to 4 will reflect success on some tasks but failure on others; these children are considered to be in the transitional stage of cognitive development and are not included in the current study.

Procedure

All students participating in the current study were required to submit an informed consent form signed by their parent. Children who agreed to participate were assured that their responses would be kept confidential and were given a sticker of their choice upon completing the assessment.

The conservation tasks were the first measures administered to each child to determine the child's cognitive developmental level. Out of 68 children screened, those who scored a total of 1 or 6 on the conservation tasks were included in the study. Next, the WJ-III fluency subtest was administered to the children to test their achievement ability. All researchers administering tests were graduate students in a Clinical Psychology, Ph.D. program and were trained before data collection began. A licensed clinical psychologist supervised the training of all researchers in the administration, scoring, and interpretation of all assessment tools. Practice assessment sessions were conducted to ensure validity and reliability of all researchers.

Results

The design was structured with one independent variable (cognitive developmental level) and three dependent variables (fluency, percentage correct and number attempted). Three one-way analyses of variance (ANOVA) were conducted to examine the specified hypothesis regarding fluency and cognition in preoperational and concrete operational children. A multivariate analysis of covariance (ANCOVA) was also conducted to rule out grade as a possible confound in the results.

The first hypothesis stated that the concrete operational children and preoperational children will differ significantly on math fluency (as measured by speed and accuracy of performing simple math calculations). An analysis of variance conducted on the math fluency subtest of the WJ-III raw scores revealed significant main effects for cognitive developmental level, $F(1, 38) = 12.82$, $p < .05$. See Table 1 for the post hoc comparison of means for fluency and cognitive developmental level concrete operational children and preoperational children. Results indicated that concrete operational children have greater math fluency than preoperational children.

The second hypothesis stated that concrete operational children and preoperational children will differ significantly on speed of performing simple math calculations. The analysis of variance conducted on the math fluency subtest of the WJ-III raw scores revealed significant main effects for cognitive developmental level, $F(1, 38) = 13.64$, $p < .05$. See Table 1 for the post hoc comparison of means for number answered and cognitive developmental level for both groups of children. Results indicated that concrete operational children answered more arithmetic problems than the preoperational children.

The third hypothesis stated that there will be no significant difference between the accuracy of concrete operational and preoperational children's math performance as measured by the percentage of simple math calculations performed correctly. An analysis of variance conducted on the math fluency subtest of the WJ-III raw scores did not reveal significant main effects for cognitive developmental level, $F(1, 38) = 2.46$, $p = .125$. See Table 1 for the post hoc comparison of means for percentage correct and the cognitive developmental level for the preoperational and concrete operational children. Results indicated that the preoperational children and the concrete operational children achieved comparable rates of accuracy.

Table 1 A comparison of mathematical calculations in preoperational and concrete operational students

Variable	Accuracy	Speed	Fluency
Concrete operational			
M	91.35	34.40	66.25
SD	11.32	8.75	18.15
N	20	20	20
Preoperational			
M	84.50	22.42	42.37
SD	15.77	11.39	23.31
N	19	19	19

Table 2 Adjusted means of an analysis of covariance of mathematical computation with grade as the covariate

Variable	Accuracy	Speed	Fluency
Concrete operational			
M	89.76	32.96	63.6
N	20	20	20
Preoperational			
M	86.17	23.92	47.3
N	19	19	19

A further analysis was performed to determine if grade confounded the results. As a result of the possible confound, an analysis of covariance was conducted with grade identified as the covariate. The ANCOVA revealed significant main effects for fluency and cognitive developmental level, $F(1, 36) = 8.65$, $p = .006$. The estimated mean for preoperational children and their fluency, $M = 47.32$ and for concrete operational children and their fluency, $M = 63.6$ (see Table 2). Results indicated that when grade is partialed from the analysis, concrete operational children have greater math fluency than preoperational children. These results are comparable with results obtained in the one-way analysis of variance conducted between cognitive ability and fluency.

An analysis of covariance was also conducted to determine the relationship between cognitive developmental level and speed (number answered) with grade identified as the covariate. The ANCOVA revealed significant main effects for number answered and cognitive developmental level, $F(1, 36) = 9.39$, $p = .004$. The estimated mean for preoperational children and their total number answered was $M = 23.93$, and for concrete operational children and their total number correct was $M = 32.97$. Results indicated that when grade is partialed from the analysis results indicated that concrete operational children answered more arithmetic problems than the preoperational children, as shown in the speed column of Table 2. These results are comparable with results obtained in the one-way analysis of variance conducted between cognitive ability and speed (number answered).

An analysis of covariance was also conducted to determine the relationship between cognitive developmental level and accuracy (percentage correct) with grade identified as the covariate. The ANCOVA revealed no significant main effects for percentage correct, $F(1, 36) = .739$, $p = .396$. The estimated mean for preoperational children and their percentage correct, $M = 86.17$ and for concrete operational children and their percentage correct, $M = 89.76$. Results indicated that when grade is partialed out from the analysis results indicated that preoperational and concrete operational children achieved comparable percentages correct (see % correct column in Table 2). These results are comparable with results obtained in the one-way analysis of variance conducted between cognitive ability and accuracy (percentage correct).

Discussion

The current study investigated the relationship between level of cognitive development and math fluency abilities in first and second grade children. Three hypotheses were evaluated to examine this relationship between fluency and cognitive ability. The first hypothesis was supported. The concrete operational children and preoperational children differed significantly on math fluency (as measured by speed and accuracy of performing simple math calculations). Concrete operational children have higher rates of fluency than the preoperational children. The results demonstrate that the children at the concrete developmental level possess both the speed and accuracy components of fluency, thus making them math fluent.

The results also supported the second hypothesis, concrete operational children and preoperational children differed significantly on their rate of speed when performing simple math calculations. The concrete operational children were able to complete more arithmetic calculations in the allotted 3 min as compared to the preoperational children. This finding suggests that concrete operational children are faster and have achieved the speed component of fluency while the preoperational children have not.

The results also supported the third hypothesis. No significant difference was found between concrete operational and preoperational children's math performance rate of accuracy. Preoperational children had the same rate of accuracy as concrete operational children when accuracy rate was determined by the percentage correct. These findings are consistent with Lovett (1987) who found that children with learning disabilities could possess the same reading accuracy as children without a learning disability, but not the same speed. Lovett's findings suggest that children are able to possess the accuracy component of fluency without the speed component. Thus, these results demonstrate that concrete operational and the preoperational children achieved comparable rates of accuracy.

The study of early mathematical development provides insights into young children's emerging academic competencies and potentially provides an empirical basis for adapting instructional methods. Because competence in arithmetic is an important goal of early schooling, research on development of arithmetic-related skills prior to and during early schooling is important for understanding and optimizing the transitions children undergo as academic knowledge is acquired (Klein and Bisanz 2000). Examining early knowledge of arithmetic principles provides important insights into children's emerging mathematical development. Identifying the mathematical relationships that children understand is consistent with calls for investigations of knowledge profiles across mathematical tasks (Bisanz and Lefevre 1992).

The results from this study suggest that a relationship exists between fluency and cognitive ability. Concrete operational children possessed both accuracy and speed (fluency) when asked to complete simple arithmetic problems. Upon further analysis, the results suggest that the concrete operational children are able to complete more problems within the specified time frame; however, the preoperational children were able to obtain comparable percentages correct. These results are important because they suggest that accuracy develops before speed. It also suggests that preoperational children do not possess math fluency.

These results indicated that preoperational children possessed comparable levels of math accuracy compared to concrete operational children. That is, preoperational children were able to obtain comparable percentages correct when completing math calculations. These results support the idea that children's individual differences need to be addressed to ensure that they receive the appropriate math instruction (Dowker 1998; Pellegrino and Goldman 1989; Widaman and Little 1992). Receiving the appropriate math instruction is crucial in the development of children's math abilities because it will allow them to enhance their performance, and possibly prevent and remediate difficulty with mathematics. Research suggests that increasing a student's accuracy and speed of responding to basic math facts is important for developing and mastering more advanced math skills (Poncy et al. 2007). Several studies have found that math fluency skills can be increased through the use of interventions that focus on training (Poncy and Skinner 2006; Poncy et al. 2007; McCallum 2006; Singer-Dudek and Greer 2005; Hartnedy et al. 2005; Sweeney et al. 2001). Poncy et al. (2007) conducted a case study comparing the effects of two interventions on basic math accuracy and fluency, Cover, Copy, and Compare (CCC) and Taped problems (TP). They found that both interventions were equally effective and increased math performance.

The results also revealed differences between preoperational and concrete operational children's math speed. The results from the study may be particularly important for understanding preoperational children in the second grade because they demonstrate that those children do not have the same speed as their concrete operational peers, thus making them achieve lower grades and possibly affecting their levels of confidence. The differences in the preoperational and concrete operational children's abilities can be reduced through appropriate intervention techniques or by creating an environment with less time restrictions. If preoperational children can improve their fluency skills they will be able to improve their math abilities overall. It is also important to provide more time to learn. Gersten et al. (2005) looked at the key findings from research on mathematics difficulties and concluded that teachers need to be aware of students who have not mastered basic combinations and provide these students with additional time to grasp concepts and operations.

Thus, the results from this study revealed that a relationship between math fluency and cognitive ability does exist and that this relationship can affect mathematical performance. The results from this study underscore how much more there is to learn about the development of children's math abilities. Specifically, future research needs to focus on the development of fluency and the relationship between fluency and cognitive abilities in other populations. Understanding the development of children's mathematical abilities will place children at an advantage and potentially optimize their transition as they develop their mathematic abilities. Limitations may have confounded these results. The number of participants and the selected population are major limitations of this study. Only 39 participants took part in this study and nearly all of the participants were from upper-middle class backgrounds. The majority of the children that participated in this study performed average to above average on the intelligence and achievement test administered as part of the larger longitudinal study. It is possible that the preoperational children's comparable rates of accuracy were due to their above average levels of intelligence. These same results may not be reproduced in children with lower levels of intelligence. Future research should examine accuracy and speed in other populations with more participants to determine if these findings can be generalized and replicated.

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