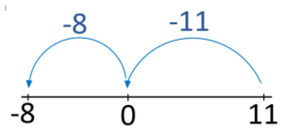
Cambridge Assessment International Education

Cambridge Lower Secondary

Cambridge Lower Secondary Mathematics 0862 Progression Grid

Number

Integers, powers and roots

Stage 7	Stage 8	Stage 9
7Ni.01 Estimate, add and subtract integers, recognising generalisations.		
Ensure learners can estimate simple calculations so that they recognise when an answer is incorrect without a formal calculation. Use numbers rounded to one significant figures to estimate. For large numbers learners can check using a calculator. Use a number line to demonstrate the relative distances from zero e.g. +2 – 2 = 0, -2 + 2 = 0, 11 - 19 = -8  -7 + 10 = ? -6 - 11 = ? -8 + 6 = ? Ensure learners understand the concept of subtracting a negative number by generalising the rule.		

7 - 2 = 5 7 - 1 = 6 7 - 0 = 7 7 - (-1) = 8 7 - (-2) = ? 7 - (-3) = ? Encourage learners to discuss calculations where a range of misconceptions have occurred. e.g. which of the following are incorrect? What mistakes have been made? 5 - 7 = 2 -5 - 4 = 9 5 - (-2) = -7		
7Ni.02 Understand that brackets, positive indices and operations follow a particular order.	**8Ni.01** Understand that brackets, indices (square and cube roots) and operations follow a particular order.	
The correct order of operations is: Brackets Indices Division/multiplication Addition/subtraction Start with any numbers inside *brackets*, going from left to right, then indices and so on. e.g. 15 - 2^2 = 15 – 4 because indices are calculated first and subtracted from 15 and not 15 – 2^2 = 13^2 Introduce examples with both addition and subtraction e.g. 9 - 3 + 4 =6 + 4 =10 **NOT** 9 – 3 + 4 = 9 - 7 = 2	The correct order of operations is: Brackets Indices, square and cube roots Division/multiplication Addition/subtraction Start with any numbers inside *brackets*, going from left to right, then indices and so on. e.g. 20 – 15 x $\sqrt{25}$ = 20 – 15 x 5 = 20 – 75 = -55 Use examples where learners have to explain the order of operations. e.g. Why are these different? A) $\sqrt{}(4^2 + 9)$ B) $(\sqrt{4} + 9)^2$	

Investigate situations where changing the operations in the question or inserting / removing brackets will change the answer. e.g. How many different answers can be found using four 4s and any combination of + - x ÷ and brackets? $(4 + 4) - 4 + 4 = 8$ Insert brackets to make this calculation correct $4^2 + 4 \times 2 = 40$		
7Ni.03 Estimate, multiply and divide integers including where one integer is negative.	**8Ni.02** Estimate, multiply and divide integers, recognising generalisations.	
Ensure learners can estimate simple calculations so that they recognise when an answer is incorrect without a formal calculation. For large numbers learners can check using a calculator. Use examples that use times tables so that learners are familiar with the numbers. e.g. $5 \times (-6) = -30$ $-30 \div 5 = -6$ At this stage do not use examples that multiply and divide negative integers by negative integers. Use calculations involving positive and negative numbers to list other calculations that can be derived e.g. If I know that $-288 \div 24 = -12$ What other calculations can I derive? ($-12 \times 24 = -228$ and so on).	Ensure learners can estimate simple calculations so that they recognise when an answer is incorrect without a formal calculation. For large numbers learners can check using a calculator. e.g. $-7 \times (-6)$; $48 \div (-8)$ Use many examples so that learners determine the rules for multiplication and division (similar to addition and subtraction) of positive and negative numbers without rote learning. Give learners a range of calculations, some of which are incorrect and ask them to identify the errors made. e.g. $-8 \times -9 = -72$ $(-10)^2 = -100$	
7Ni.04 Understand lowest common multiple and highest common factor (numbers less than 100).	**8Ni.03** Understand factors, multiples, prime factors, highest common factors and lowest common multiples.	
Include examples where a listing strategy can be used. e.g.	e.g. Construct a factor tree and write a number as a product of its prime factors e.g. $500 = 2^2 \times 5^3$	

<table>
<tr>
<td>The multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24…
The multiples of 4 are 4, 8, 12, 16, 20, 24 …
The common multiples of 3 and 4 are 12, 24, …
The lowest common multiple of 3 and 4 is 12.
Note that the Highest Common Factor (HCF) is the same as the Greatest Common Divisor.
e.g.
Factors of 36 are:
1, 2, 3, 4, 6, 9, 12, 18, 36
Factors of 48 are:
1, 2, 3, 4, 6, 8, 12, 16, 24, 48
The HCF of 36 and 48 is 12.</td>
<td>Use prime factorisation to find the LCM or HCF of two numbers
e.g.
40 = 2 x 2 x 2 x 5
28 = 2 x 2 x 7
LCM = 2 x 2 x 2 x 5 x 7 = 280
HCF = 2 x 2 = 4</td>
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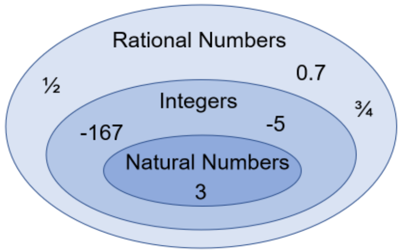
</tr>
<tr>
<td>7Ni.05 Use knowledge of tests of divisibility to find factors of numbers greater than 100.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ensure learners are aware of the divisibility rules for numbers between 2 and 10
<table>
<tr><th>Test for</th><th>Divisibility test</th><th>Example</th></tr>
<tr><td>2</td><td>Last digit must be 0, 2, 4, 6 or 8</td><td>456</td></tr>
<tr><td>3</td><td>Sum of the digits must be divisible by 3</td><td>291
(2+9+1 = 12)</td></tr>
<tr><td>4</td><td>Number formed by the last two digits must be divisible by 4</td><td>8916</td></tr>
<tr><td>5</td><td>Last digit must be 0 or 5</td><td>3875</td></tr>
<tr><td>6</td><td>Number must be divisible by 2 and 3</td><td>162
1+6+2=9</td></tr>
<tr><td>7</td><td>Number formed when the last digit is doubled and taken away from the remaining number must be divisible by 7</td><td>133
13 –(3x2) = 7</td></tr>
<tr><td>8</td><td>Number formed by the last three digits must be divisible by 8</td><td>5720</td></tr>
<tr><td>9</td><td>Sum of the digits must be divisible by 9</td><td>378
(3+7+8 =18)</td></tr>
<tr><td>10</td><td>Last digit must be 0</td><td>30</td></tr>
</table>
25 – last two digits are 00, 25, 50 or 75
100 – last 2 digits are 00
e.g. The number 162 is divisible by 9 and also divisible by 3 because 1 + 6 + 2 = 9 and nine is divisible by 3.</td>
<td></td>
<td></td>
</tr>
</table>

<table>
<tr><th></th><th>8Ni.04 Understand the hierarchy of natural numbers, integers and rational numbers.</th><th>9Ni.01 Understand the difference between rational and irrational numbers.</th></tr>
<tr><td></td><td>
Learners should be able to represent the hierarchy in a Venn diagram.



Ask questions such as:
Write down a number from the list “-4, 0.5, 16.3, π, 10,000” that is:
a) a rational number
b) an integer
c) a natural number

Tick all the correct statements:
a) 3 is a rational number
b) -3 is a rational number
c) 3 is a natural number
d) -3 is a natural number
e) 3 is an integer
f) -3 is an integer
</td><td>
Ensure learners understand that integers and natural numbers are rational and that fractions (with integer numerator and denominator), terminating decimals or recurring decimals are also rational.
Ensure learners understand that some roots (e.g. $\sqrt{2}, \sqrt{3}, \sqrt{5}$) are irrational but some roots are rational (e.g. $\sqrt{4}$ is rational as it is equal to 2). Ensure learners understand that π is an example of an irrational number.
Learners are expected to understand that negative roots do not belong to the set of rational or irrational numbers.
Ask questions such as:
Circle the number from the list below that is irrational:
-4, 0.5, 16.3, π, 10,000, $\sqrt{25}$
Circle all the rational numbers:
-4, 0.5, π, $\sqrt{2}$, 10, ½

Tick the correct boxes
<table>
<tr><th></th><th>Rational</th><th>Irrational</th></tr>
<tr><td>5</td><td></td><td></td></tr>
<tr><td>$\sqrt{25}$</td><td></td><td></td></tr>
<tr><td>$\sqrt{2}$</td><td></td><td></td></tr>
<tr><td>½</td><td></td><td></td></tr>
</table>
Explore connections between rational and irrational numbers
Ask learners to find: two irrational numbers whose product is rational
e.g. use a calculator to work out
$\sqrt{2} \times \sqrt{8}$
$\sqrt{3} \times \sqrt{12}$
</td></tr>
</table>

	8Ni.05 Use positive and zero indices, and the index laws for multiplication and division.	**9Ni.02** Use positive, negative and zero indices, and the index laws for multiplication and division.
	e.g. know that $7^0 = 1$, $7^2 \times 7^3 = 7^5$, $(15^3)^2 = 15^6$ $11^{14} \div 11^{12} = 11^2$	e.g. know that $2^{-3} = \frac{1}{2^3}$, $11^{10} \div 11^{12} = 11^{-2}$
		9Ni.03 Understand the standard form for representing large and small numbers.
		Standard form is the same as scientific notation. $2530000 = 2.53 \times 10^6$ and not 253×10^4 or 25.3×10^5 $0.00568 = 5.68 \times 10^{-3}$
7Ni.06 Understand the relationship between squares and corresponding square roots, and cubes and corresponding cube roots.	**8Ni.06** Recognise squares of negative and positive numbers, and corresponding square roots.	**9Ni.04** Use knowledge of square and cube roots to estimate surds.
Positive squares only. Ensure learners can estimate simple calculations so that they recognise when an answer is incorrect without a formal calculation. Learners can check using a calculator where appropriate. Ensure learners use the notation for square, cube and square root. Ensure learners recognise the relationship between square and square roots (and cube and cube roots) as inverse operations. e.g. $25^2 = 625$ and $\sqrt{625} = \sqrt{25^2} = 25$. $3^3 = 27$ so $\sqrt[3]{27} = 3$	Ensure learners can estimate simple calculations so that they recognise when an answer is incorrect without a formal calculation. Learners can check using a calculator where appropriate. Ensure learners recognise that the square root of a number may have positive or negative answers e.g. The square root of 25 is 5 or -5 because $5 \times 5 = 5^2 = 25$ and $(-5) \times (-5) = (-5)^2 = 25$	Ensure learners can estimate simple calculations so that they recognise when an answer is incorrect without a formal calculation. Learners can check using a calculator where appropriate. e.g. recognise that $\sqrt{9} < \sqrt{10} < \sqrt{16}$ so $3 < \sqrt{10} < 4$ Ensure learners know that when the answer to a square or cube root is irrational (e.g. $\sqrt{5} = 2.223606\ldots$ or $\sqrt[3]{30} = 3.107232\ldots$) it is referred to as a surd.
	8Ni.07 Recognise positive and negative cube numbers, and the corresponding cube roots.	
	Ensure learners understand the equivalence of the notation $\sqrt[3]{-64} = \sqrt[3]{(-4)^3} = \sqrt[3]{(-4) \times (-4) \times (-4)} = -4$	

Number

Place value, ordering and rounding

Stage 7	Stage 8	Stage 9
7Np.01 Use knowledge of place value to multiply and divide whole numbers and decimals by any positive power of 10.	**8Np.01** Use knowledge of place value to multiply and divide integers and decimals by 0.1 and 0.01.	**9Np.01** Multiply and divide integers and decimals by 10 to the power of any positive or negative number.
e.g. $5 \times 10^5 = 500\ 000$ $7 \div 10^3 = 0.007$ Ensure learners recognise the notation for positive powers of 10 $10^0 = 1$ $10^1 = 10$ $10^2 = 100$ $10^3 = 1\ 000$ $10^4 = 10\ 000$ $10^5 = 100\ 000$ etc.	e.g. $0.235 \div 0.1 = 2.35$ $0.235 \times 0.1 = 0.0235$ Ensure learners understand the position of the decimal point when multiplying and dividing by $\frac{1}{10}$s (tenths). When dividing by 0.1 or 0.01 consider equivalent calculations 3.5 ÷ 0.1 is the same as 35 ÷ 1 43.7 ÷ 0.01 is the same as 437 ÷ 0.1 and 4 370 ÷ 1	e.g. $2.4 \times 10^5 = 240\ 000$ $5.82 \times 10^{-3} = 0.00582$ Ensure learners understand the different representations of powers of ten: $0.1 = \frac{1}{10} = 10^{-1}$; $0.01 = \frac{1}{100} = \frac{1}{10}^2 = 10^{-2}$
7Np.02 Round numbers to a given number of decimal places.	**8Np.02** Round numbers to a given number of significant figures.	**9Np.02** Understand that when a number is rounded there are upper and lower limits for the original number.
e.g. round 5.6470299 to 2 decimal places (d.p.) (5.65) Use examples that give solutions to problems with an appropriate degree of accuracy. Learners can use calculators to find the answer to a question and then round the answer.	e.g. round 34 627 to 2 significant figures (35 000) Round 2.8963 to 3 significant figures (2.90) With the number 593 247 the 5 is the most significant digit, because the number is 5 hundred thousand and something.	If a number was rounded to the nearest 10 to be 460, learners understand that the original number could be any number between 455 (including) and 465 (excluding). Ensure learners can use inequality notation to describe the limits (bounds) on the original number, x. e.g. $455 \le x < 465$

e.g. They use the calculator to find $\sqrt{62} = 7.874007874\cdots$ and round to 2 decimal places to get to 7.87	Then 9 is the next most significant, and so on. 593 247 correct to 1 significant figure is 600 000 With the number 0.0000057263, the 5 is the most significant digit, because it tells us that the number is 5 millionths and something. The 7 is the next most significant, and so on. 0.0000057263 rounded to 2 significant figures is 0.0000057 Use examples that give solutions to problems with an appropriate degree of accuracy Use examples where zero is used as a place holder e.g. Round 20 435 to 3 s.f. — 20 400 0.2056 to 2 s.f. — 0.21 40 505 to 2 s.f. — 41 000	*This will assist learners in their preparation for understanding and using bounds at IGCSE.*

Number

Fractions, decimals, percentages, ratio and proportion

<table>
<tr><th>Stage 7</th><th>Stage 8</th><th>Stage 9</th></tr>
<tr><td>7Nf.01 Recognise that fractions, terminating decimals and percentages have equivalent values.</td><td>8Nf.01 Recognise fractions that are equivalent to recurring decimals.</td><td>9Nf.01 Deduce whether fractions will have recurring or terminating decimal equivalents.</td></tr>
<tr><td>
Ensure learners understand that:

a percentage can be written as a fraction out of 100
place value can help to convert decimals into fractions with a denominator of 10, 100, 1000 and vice versa.

e.g. 31% = $\frac{31}{100}$ = 0.31
0.7 = $\frac{7}{10}$ = 70%
0.43 = $\frac{43}{100}$ = 43%
0.047 = $\frac{47}{1000}$ = 4.7%
Use examples where learners are required to show the equivalence between fractions, decimals and percentages
<table>
<tr><th>F</th><th>D</th><th>P</th></tr>
<tr><td>$\frac{1}{2}$</td><td></td><td></td></tr>
<tr><td></td><td>0.25</td><td></td></tr>
<tr><td></td><td></td><td>75%</td></tr>
<tr><td>$\frac{1}{10}$</td><td></td><td></td></tr>
<tr><td></td><td>0.3</td><td></td></tr>
<tr><td>$\frac{1}{5}$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>23%</td></tr>
<tr><td></td><td>0.31</td><td></td></tr>
<tr><td></td><td></td><td>13.2%</td></tr>
</table>
Ensure learners can interpret fractions as a division.
e.g. $\frac{3}{8}$ can be converted to a decimal by dividing 3 by 8 (3 ÷ 8 = 0.375)
</td><td>
Ensure learners can interpret fractions as division and that some decimal representations of fractions will terminate and some will recur.
e.g. $\frac{3}{4}$ = 3 ÷ 4 =0.75 so this is terminating
$\frac{1}{3}$ = 1 ÷ 3 = 0.33333… $0.\dot{3}$ so this is recurring
Encourage learners to work systematically to convert unit fractions to decimal equivalents using a written method so they can notice patterns
<table>
<tr><th>F</th><th>D</th></tr>
<tr><td>$\frac{1}{2}$</td><td></td></tr>
<tr><td>$\frac{1}{3}$</td><td></td></tr>
<tr><td>$\frac{1}{4}$</td><td></td></tr>
<tr><td>$\frac{1}{5}$</td><td></td></tr>
<tr><td>$\frac{1}{6}$</td><td></td></tr>
<tr><td>$\frac{1}{7}$</td><td></td></tr>
<tr><td>$\frac{1}{8}$</td><td></td></tr>
<tr><td>$\frac{1}{9}$</td><td></td></tr>
<tr><td>$\frac{1}{10}$</td><td></td></tr>
<tr><td>$\frac{1}{11}$</td><td></td></tr>
<tr><td>$\frac{1}{12}$</td><td></td></tr>
</table>
</td><td>
e.g.
Learners know
$\frac{1}{9} = 0.\dot{1}$
So, $\frac{4}{9} = 4 \times \frac{1}{9} = 4 \times 0.\dot{1} = 0.\dot{4}$
Ensure learners know that if 3 is a factor of the denominator of the fraction (when in its simplest form) then it will result in a recurring decimal.
e.g. $\frac{1}{9}$ must be recurring as
$\frac{1}{3}$ = $0.\dot{3}$ and because
$\frac{1}{3}$ = $\frac{3}{9}$ = 3 x $\frac{1}{9}$
Consider which denominators result in recurring decimals.
e.g.
If the denominator is a multiple of 7 will it recur?
Does $\frac{5}{14}$ recur when converted to a fraction?
What about $\frac{7}{14}$?
</td></tr>
</table>

Use examples that include improper fractions and mixed numbers. 1.245 = 124.5% = $\frac{249}{200}$	Use examples where learners convert non-unit fractions to decimals e.g. $\frac{3}{7}$ 3÷7 = 0.428571…	
7Nf.02 Estimate and add mixed numbers, and write the answer as a mixed number in its simplest form.	**8Nf.02** Estimate and subtract mixed numbers, and write the answer as a mixed number in its simplest form.	**9Nf.02** Estimate, add and subtract proper and improper fractions, and mixed numbers, using the order of operations.
Use examples that add mixed numbers with the same and different denominators. Learners can use any method but should consider which is most efficient. e.g. Learners understand when the most effective point to regroup numbers is. $3\frac{4}{5} + 2\frac{3}{10} = 3\frac{8}{10} + 2\frac{3}{10} = 5\frac{11}{10} = 6\frac{1}{10}$ require regrouping of the mixed number rather than converting to improper fractions. Learners express answers greater than one as a mixed number, and reduce all fractions to their simplest form.	Use examples that subtract mixed numbers with the same and different denominators. e.g $2\frac{3}{7} - 1\frac{1}{5}$ Learners can use any method but should consider which is most efficient. e.g. learners understand that they need to regroup the first number. $3\frac{1}{5} - 2\frac{3}{5} = 2\frac{6}{5} - 2\frac{3}{5} = \frac{3}{5}$ Learners express answers greater than one as a mixed number, and reduce all fractions to their simplest form.	e.g. in $1\frac{3}{8} - \left(\frac{2}{5} + \frac{1}{2}\right)$ we calculate the fractions in the brackets first $1\frac{3}{8} - (\frac{9}{10}) = \frac{55}{40} - \frac{36}{40} = \frac{19}{40}$ Ensure learners are also able to apply learning from **9Nf.03** to use the order of operations with multiplication and division as well as addition and subtraction. e.g. $5\frac{3}{7} - 1\frac{1}{5} \times \frac{7}{2}$ We calculate the multiplication first $5\frac{3}{7} - 4\frac{1}{5} =$ $1\frac{8}{35}$ e.g. $\left(1 - \frac{3}{4}\right) \div \left(1 - \frac{1}{8}\right)$
7Nf.03 Estimate, multiply and divide proper fractions.	**8Nf.03** Estimate and multiply an integer by a mixed number, and divide an integer by a proper fraction.	**9Nf.03** Estimate, multiply and divide fractions, interpret division as a multiplicative inverse, and cancel common factors before multiplying or dividing.

e.g. $\frac{7}{12} \times \frac{4}{12} = \frac{28}{144}$ Learners should be able to simplify answers although some learners may do this in stages. e.g. $\frac{28}{144} = \frac{14}{72} = \frac{7}{36}$ Learners become aware that fractions like $\frac{4}{12}$ could be simplified before the fractions are multiplied and once fluent, learners may cross cancel before multiplying to improve efficiency. e.g. $\frac{7}{12} \times \frac{4}{9} = \frac{7}{3} \times \frac{1}{9} = \frac{7}{27}$ For multiplication: Ensure learners understand that, when an operator is less than one (proper fraction) the answer will be smaller than the original quantity. e.g. $\frac{3}{4}$ x $\frac{1}{6}$ = $\frac{1}{8}$ and $\frac{1}{8}$ is smaller than $\frac{3}{4}$ and $\frac{1}{6}$ For division: Ensure learners appreciate that for example dividing by $\frac{1}{3}$ is the same as multiplying by 3 and develop the idea that for example dividing by $\frac{2}{3}$ is the same as multiplying by $\frac{3}{2}$. Learners understand that dividing by a fraction is the same as multiplying by its reciprocal. Ensure learners understand that, when an operator is less than one (proper fraction) the answer will be bigger than the original quantity. e.g. $\frac{5}{8} \div \frac{3}{4} =$ 5/ $\frac{5}{8} \times \frac{4}{3} = \frac{20}{24} = \frac{5}{6}$	Learners should be able to multiply integers by mixed numbers by partitioning e.g. $20 \times 1\frac{3}{8}$, 20 x 1 = 20 $20 \times \frac{3}{8} = \frac{60}{8} = 7\frac{4}{8} = 7\frac{1}{2}$ $20 + 7\frac{1}{2} = 27\frac{1}{2}$ Learners should also understand that converting the mixed number to an improper fraction before multiplying will result in the same answer. Learners should be able to divide integers by unit fractions initially, understanding that, for example, dividing an integer by $\frac{1}{2}$ is the same as multiplying by 2 as they are counting up how many halves are needed to make that integer. e.g. $5 \div \frac{1}{2} = 5 \times 2 = 10$ $7 \div \frac{1}{9} = 7 \times 9 = 63$ Learners should know that when they divide A by B it means how many lots of B are in A, rather than systematically learning a rule to calculate the answer. e.g. $12 \div \frac{3}{4}$ means how many $\frac{3}{4}$ are there in 12. The answer is 16 Learners should understand that dividing an integer by a fraction is the same as multiplying by its reciprocal. e.g. $12 \div \frac{3}{4} = 16$ $12 \times \frac{4}{3} = \frac{48}{3} = 16$	e.g. $2\frac{1}{5} \times \frac{13}{22} =$ $\frac{11}{5} \times \frac{13}{22} = \frac{1}{5} \times \frac{13}{2} =$ $\frac{13}{10} = 1\frac{3}{10}$ Use examples that use a combination of proper and improper fractions with mixed numbers or integers. Learners will need to be able to change mixed numbers into improper fractions e.g. $1\frac{1}{4} \div 3\frac{2}{5}$ Ensure learners understand that proper and improper fractions, and mixed numbers can act as operators that either increase or decrease the original value. e.g. when a quantity is multiplied by a mixed number/improper fraction then the answer is larger than the original quantity. When a quantity is divided by a mixed number/improper fraction then the answer is smaller than the original quantity. e.g. $20 \times 1\frac{1}{2} = 30$ $20 \div 1\frac{1}{2} = 13\frac{1}{3}$

and $\frac{5}{6}$ is bigger than $\frac{5}{8}$		
7Nf.04 Use knowledge of common factors, laws of arithmetic and order of operations to simplify calculations containing decimals or fractions.	**8Nf.04** Use knowledge of the laws of arithmetic and order of operations (including brackets) to simplify calculations containing decimals or fractions.	**9Nf.04** Use knowledge of the laws of arithmetic, inverse operations, equivalence and order of operations (brackets and indices) to simplify calculations containing decimals and fractions.
Encourage learners to work mentally where appropriate to select an efficient strategy for the problem. Use examples with decimals only or proper fractions only, not combined, and of multiplying decimals by integers (not decimal x decimal) e.g. 0.5 x 7 x 10 = 0.5 x 10 x 7 =5 x 7 = 35 4.7 × 9 = 4.7 × (10 – 1) = 47 – 4.7 = 42.3 $\frac{3}{5}$ x $\frac{2}{3}$ + $\frac{4}{5}$ = $\frac{11}{5}$	Encourage learners to work mentally where appropriate to select an efficient strategy for the problem. Use examples - with decimals only or fractions only, not combined. $\frac{3}{5}$ x ($\frac{2}{3}$ + $\frac{4}{5}$) = $\frac{66}{75}$ = $\frac{22}{25}$ - that use one or more of the four operations. (0.3 + 0.1) x 1.2=0.48 - of multiplying decimals by decimals. 4 × 7.6 × 2.5 = 4 × 2.5 × 7.6 = 10 × 7.6 = 76 4.7 × 9.9 = 4.7 × (10 – 0.1) = 47 – 0.47 = 46.53	Encourage learners to work mentally where appropriate to select an efficient strategy for the problem. Use examples that use one or more of the four operations and a mix of fractions and decimals e.g. $\left(\frac{1}{2}+6.5\right)^2$ –1 = 48 2.5 × 2.5 × 12 = $\frac{5}{2} \times \frac{5}{2} \times 12$ = 5 × 5 × 3 = 75 3.64 × 5^2 = $\frac{364}{100} \times 25 = \frac{364}{100} \times \frac{100}{4}$ = 364 ÷ 4= 91
7Nf.05 Recognise percentages of shapes and whole numbers, including percentages less than 1 or greater than 100.	**8Nf.05** Understand percentage increase and decrease, and absolute change.	**9Nf.05** Understand compound percentages.
e.g. $\frac{1}{2}$% is half of 1% $\frac{1}{2}$% of 400 = 2 e.g. finding $\frac{1}{2}$% or 0.5% of a quantity can be confused with finding 50% e.g. 2 $\frac{1}{2}$ = 2.5 = 250% Express any quantity as a percentage of another.	e.g. increase \$300 by 20% Ensure learners can use multipliers to increase and decrease by a percentage Increase \$300 by 20% \$300 x 1.2=\$360 Decrease £200 by 15% £200 x 0.85 = £170	e.g. If an item priced at \$200 has a 10% increase in price then the new price is \$220. If this price is followed by a 10% decrease (a decrease of \$22), the final price will be \$198, *not* the original price of \$200. e.g. If a new car which costs \$11,000 decreases in price by 10% each year, how many years will it be before the car is worth less than \$6,000? What situations could these calculations represent? A) 600 x 1.04^7

If I have a block of wood that is 25 centimetres long and I cut a piece from it that is 19 centimetres long, what percentage did I cut? What percentage will be left? Understand the multiplicative relationship of percentage e.g. 300% is an increase by a factor of 3, 300% of 50 is 150 (which is a 200% increase) If the price of a packet of sweets increases from \$1.50 to \$4.50 then this is 300% of the original amount since this is the new price is three times the original price and this represents a 200% increase. Ensure learners understand that if the price of an item increases by 10% then the final price is 110% of the initial price (100% + 10% = 110%)	Use examples that require learners to understand percentage decrease and absolute change e.g. 120 decreased by 200% = -120 and absolute change is -120 -120 = - 240 An increase of 800% means the final amount is 9 times the original (100% + 800% = 900% = 9 times as large).	B) 800 x 0.95^3
7Nf.06 Understand the relative size of quantities to compare and order decimals and fractions, using the symbols =, ≠, > and <.	**8Nf.06** Understand the relative size of quantities to compare and order decimals and fractions (positive and negative), using the symbols =, ≠, >, <, ≤ and ≥.	
e.g. order proper fractions and improper fractions $\frac{3}{4}, \frac{2}{3}, \frac{4}{3}$ and $\frac{3}{2}$ e.g. order from smallest to largest $\frac{3}{5}, \frac{11}{20}, \frac{5}{8}$ and $\frac{8}{15}$ Using the symbols =, ≠, > and < ask learners to order and compare values. e.g. given that 3.164 < *x* < 3.167 write down a possible value for *x*. e.g. given that $3\frac{1}{5} < x < 3\frac{1}{4}$ write down a possible value for *x*.	e.g. compare a mark of 43 out of 55 to a mark of 80% Using the symbols =, ≠, >, <, ≤ and ≥ ask learners to order and compare values. e.g. Understand that $4.17 \leq x < 4.34$ x represents any value in the range 4.17 inclusive to 4.34 exclusive. Ensure learners understand that when they are ordering and comparing fractions: • For the same denominator, the larger the numerator the larger the fraction. • For the same numerator, the larger the denominator the smaller the fraction e.g. insert the correct symbol (< or >) $\frac{7}{11}$ $\square$ $\frac{8}{11}$	

	<table><tr><td>$\frac{23}{37}$</td><td></td><td>$\frac{23}{41}$</td></tr></table> Ensure learners understand that to compare $\frac{37}{65}$ with $\frac{48}{4}$ decimals or percentages are more efficient than fractions. e.g. decide the better buy between a 500 g packet of rice at \$0.35, or a packet with 500 g + 25% extra at \$0.39	
7Nf.07 Estimate, add and subtract positive and negative numbers with the same or different number of decimal places.		
e.g. Three children took part in a long jump competition. John jumped 1.2m, Mary jumped 0.15m further than John, Fatima jumped 0.3m less than Mary. How far did Fatima jump? Ensure learners can calculate with negative numbers and decimals e.g. –7.18 + 3.9 1.208 – (0.7 + 2.41)		
7Nf.08 Estimate, multiply and divide decimals by whole numbers.	**8Nf.07** Estimate and multiply decimals by integers and decimals.	**9Nf.06** Estimate, multiply and divide decimals by integers and decimals.
e.g. 13.27 × 108 4.132 ÷ 5 5.025 ÷ 25 Ensure learners can continue a division to a specified number of decimal places. e.g. Write 12.55 ÷ 7 to 1 d.p 12.55 ÷ 7 = 1.79… So 12.55 ÷ 7=1.8 to 1d.p.	Use examples with positive decimals and positive and negative integers. e.g. -2 x 0.4 = -0.8 0.45 x 0.56 Consider patterns to develop an understanding of where to place the decimal point e.g. 0.2 x 3 = 0.6 0.2 x 0.3 = 0.06	Use examples with both positive and negative decimals and integers. e.g. • 2.34 × 2.9 • 43.32 ÷ 0.12 • –2.7 ÷ 3 • –3 × –6 × –0.5 e.g. If we know that 2.88÷0.24=12 What other calculations can you deduce?

	0.2 x 0.03 = 0.006	(12 x 0.24 = 2.88 and so on) Ensure learners recognise the effects of multiplying and dividing by decimals between 0 and 1. e.g. Learners should be able to recognise, without doing a calculation, whether the answer to 3.784 ÷ 0.916 will be bigger or smaller than 3.784
	8Nf.08 Estimate and divide decimals by numbers with one decimal place.	
	Use examples with positive decimals and positive and negative integers. e.g. -2 x 0.4 = -0.8 4 ÷ 0.5 = 8 0.45 x 0.56 5.21 ÷ 0.2 Consider equivalent calculations to develop understanding e.g. 4.8 ÷ 0.6 is equivalent to 48 ÷ 6	
7Nf.09 Understand and use the unitary method to solve problems involving ratio and direct proportion in a range of contexts.	**8Nf.09** Understand and use the relationship between ratio and direct proportion.	**9Nf.07** Understand the relationship between two quantities when they are in direct or inverse proportion.
Ensure learners understand that once the value of 1 unit is known, the value of multiple units can be found by multiplying e.g. if 12 tins of paint weigh 30kg, how much will 5 tins weigh? The first step in solving this is to find what ONE tin weighs. This will be $\frac{30}{12}$ so 2.5kg. Scaling this back up for 5 tins gives 5 x 2.5 = 12.5kg Include examples in context. e.g. - Recipes:	When A and B share money in the ratio 4:3, it means that A gets $\frac{4}{7}$ of the money and B gets $\frac{3}{7}$ Ensure learners can express a given ratio as proportion. e.g. The ratio of red to blue counters in a bag is 5:4 What proportion are red? ($\frac{5}{9}$) Hannah says she thinks there are 64 counters in the bag altogether. Is she correct?	Ensure learners can identify and give examples of both direct and indirect proportion and understand the difference between them. Use simple examples of inverse proportion. e.g. An athlete runs a race and takes 120 seconds to finish. If he runs double the speed, how long will it take him? e.g. It takes 12 builders 2 weeks to build a garage. How long would it take 6 builders to do the same job?

300g of sugar makes 12 biscuits. How much sugar is needed for 5 biscuits? - Currencies: David exchanged 350 Euros and received $400. How much is 1 Euro worth in dollars? Ben received $6000. How many Euros did he pay?	Ensure learners can express given proportions as a ratio. e.g. There are some blue and red beads on a bracelet. $\frac{2}{5}$ of the beads are blue. What is the ratio of blue to red beads? (2:3) Ensure learners can also use equivalence to compare proportions. e.g. A has yellow and red sweets in the ratio 20:480 B has yellow and red sweets in the ratio 15:385 Who has the greater proportion of yellow sweets?	
7Nf.10 Use knowledge of equivalence to simplify and compare ratios (same units).	**8Nf.10** Use knowledge of equivalence to simplify and compare ratios (different units).	
Ensure learners understand that the simplest form contains no decimals or fractions so 1:1.5 in its simplest form is 2:3 Ensure learners understand that when simplifying a ratio the relationship between the parts of the ratio does not change but the quantities may increase or decrease. e.g. 10:25 simplifies to 2:5 If 10 changes to 2 (5 times less) then 25 must change to 5 in the same relationship (5 times less). Use examples with the same units e.g. cm and cm A model of a house is 30cm tall. The house is 500cm tall. Write the ratio of the model height to the house height in its simplest form. 30cm:500cm 3:50 Ensure learners can use equivalence to compare ratios to solve a problem. e.g. 30 biscuits cost $1	Use examples with different units e.g. m and cm A model of a house is 30cm tall. The house is 5m tall. Write the ratio of the model height to the house height in its simplest form. 30cm:5m 30cm:500cm 3:50 Ensure learners can use equivalence to compare ratios to solve a problem including different units of measure. e.g. 3 cups hold 1300ml 4 mugs hold 1.5L Which will hold more liquid, one cup or one mug?	

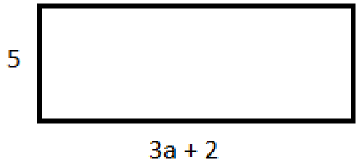
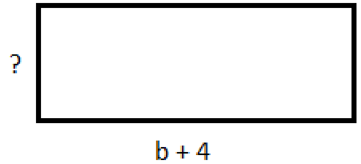
45 biscuits cost $1.80 Which is best value? 30:$1 is the same as 15:50p 45:$1.80 is the same as 15:60c		
7Nf.11 Understand how ratios are used to compare quantities to divide an amount into a given ratio with two parts.	**8Nf.11** Understand how ratios are used to compare quantities to divide an amount into a given ratio with two or more parts.	**9Nf.08** Use knowledge of ratios and equivalence for a range of contexts.
Ensure learners can share a quantity into a given ratio (two parts). e.g. Share 70 sweets between A and B, in the ratio 6:4 Use a table or diagram to assist learners. A 6 42 B 4 28 Total 10 70 Consider the fraction of the sweets each person will get A will get $\frac{6}{10}$ $(\frac{42}{70})$ and B will get $\frac{4}{10}$ $(\frac{28}{70})$.	Use examples of simplifying ratios with more than 2 parts. e.g. 18:27:63 simplified to 2:3:7 Ensure learners can share a quantity into a given ratio (include examples with more than two parts). E.g., the angles in a triangle are in the ratio 2:7:9. Find the size of the smallest angle.	Use examples of sharing in a ratio where learners have to work backwards and use knowledge of the relative size of the parts of a ratio. e.g. Some sweets are shared in the ratio 5:4:2 between A, B, and C. B gets 48. How many sweets were there altogether? e.g. A & B share money in the ratio 8:5. A gets $21 more than B. Find out how much B gets. e.g. Two numbers are in the ratio 5 : 2 one of the numbers is 0.8 find two possible values for the other number.

Algebra

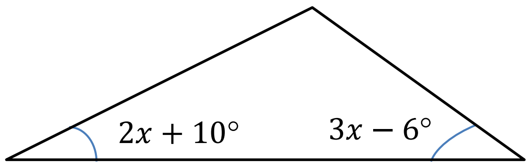
Expressions, equations and formulae

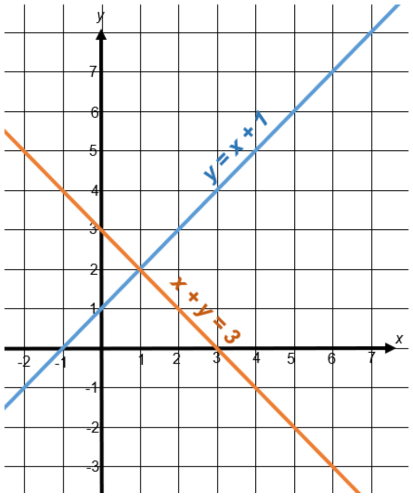
Stage 7	Stage 8	Stage 9
7Ae.01 Understand that letters can be used to represent unknown numbers, variables or constants.	**8Ae.01** Understand that letters have different meanings in expressions, formulae and equations.	
Ensure learners understand the meaning of unknown, constant, coefficient and term. e.g. in $3x - 2 = 7$, x is the unknown, 3 is the coefficient of x, 2 and 7 are constants, $2n$ is a term, $3n + 5$ is an expression containing two terms ($3n$ and 5), $4n - 5 = 11$ is an equation containing an unknown n and there is only one possible value of n. In some cases, constants are given as numbers and in other cases as letters because they represent a relationship. e.g. • constant as numbers in $5x + 9 = 17$, the constants are 9 and 17 • constant as letters π is the ratio of a circle's circumference to its diameter ≈ 3.14 Use examples that relate to real situations • John is x years old, Anne is y years old. What might the expression x + y mean? • What would the expression 2x mean? • What would the expression x - y mean? Is this the same as y - x? • What could the equation x + 3 = y mean?	Ensure learners understand the difference between a variable and an unknown, and the meaning of formula and subject of a formula. e.g. • In the formula $v = u + at$, the letters are variables related by the formula. v is the subject of the formula. • $y = 5x + 4$ is a formula but also a linear function in two variables (x and y). For each individual value of x, the value of y can be calculated (y varies according to the value of x) • In the linear equation $4n - 5 = 11$, there is only one possible value of n. n is an "unknown" but the equation can be solved to find what n is. • In the expression $5x + 4$, x is a variable, 4 is a constant, $5x$ and 4 are terms of the expression. 5 is the coefficient of x. The expression is not equal to anything so cannot be solved. The emphasis in Stage 8 is on the difference between equations, expressions and formulae. e.g. which of the following is an expression? V=IR 3x + 4 = 19 10p + 15 Which one is an equation?	

7Ae.02 Understand that the laws of arithmetic and order of operations apply to algebraic terms and expressions (four operations).	**8Ae.02** Understand that the laws of arithmetic and order of operations apply to algebraic terms and expressions (four operations, squares and cubes).	**9Ae.01** Understand that the laws of arithmetic and order of operations apply to algebraic terms and expressions (four operations and integer powers).
e.g. in $5+\frac{a}{2}$ the division is calculated first; in $9-4b$ the multiplication is calculated first $a + b = b + a$ (commutative) $ab = ba$ Use examples with no brackets e.g. Find the value of $5+\frac{a}{2}$ when $a=6$, $5+\frac{6}{2}$ $=5+3$ =8 Find the value of 3a + 5 when a = 7 Find the value of a + bc when a = 3, b = 1 and c = 10	Examples can include brackets e.g. In $2a^3$ the power is calculated first; in $4(d-3)$ the bracket is calculated first; in $\frac{3a+5}{2}$ the numerator is calculated first. e.g. Find the value of $3x^2+4$ when $x=-2$, $3(-2)^2+4$ $=3(4)+4$ =12+4 =16 e.g. Calculate the following if a=3, b=5 and c=6 • a(b + c) • ab + c • $(a + b)^2$ • $a^2 + b^2$	Examples can include brackets and all operations e.g. In the expression, $3(x+8)-\frac{x^2}{4}+5$, the brackets are solved first, then the power of two, then the multiplication and division, and then the addition and subtraction from left to right. In order to solve this expression for $x=2$, following the order of operations: $3(2+8)-\frac{2^2}{4}+5$ $3(10)-\frac{2^2}{4}+5$ $3(10)-\frac{4}{4}+5$ $30-1+5$ $29+5=34$
7Ae.03 Understand how to manipulate algebraic expressions including: - collecting like terms - applying the distributive law with a constant.	**8Ae.03** Understand how to manipulate algebraic expressions including: - applying the distributive law with a single term (squares and cubes) - identifying the highest common factor to factorise.	**9Ae.02** Understand how to manipulate algebraic expressions including: - expanding the product of two algebraic expressions - applying the laws of indices - simplifying algebraic fractions.
Use and interpret algebraic notation (convention), including: • $3y$ in place of $y+y+y$ and $3\times y$ and y3 • ab in place of $a\times b$ • a^2 in place of $a\times a$, a^3 in place of $a\times a\times a$; a^2b in place of $a\times a\times b$ • $\frac{a}{b}$ in place of $a\div b$ Collect like terms $a+a+a+a=4a$	Expanding Distributive law with a single term $3a(a-3) = 3a^2-9a$ $5b^2(b-2a) = 5b^3-10ab^2$ Ensure learners understand that factorising means identifying common factors in algebraic expressions. Link with multiples and factors and prime numbers.	Expanding two brackets to form trinomials where the coefficient of x is 1: $(x+3)(x+2)=x^2+5x+6$ $(y-1)(y+3)=y^2+2y-3$ Ensure learners recognise the generalisations: $(x+a)(x-a)=x^2-a^2$ (difference of two squares) $(x+a)^2=x^2+2ax+a^2$

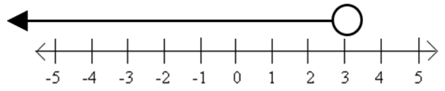
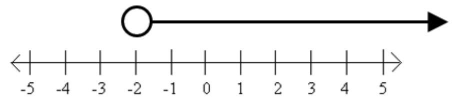
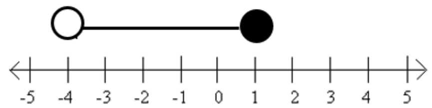
$3a^2 - a^2 + 4a^2 = 6a^2$ $3c + 5 - 2c = c + 5$ Include examples with fractional coefficients $3b + \frac{b}{2} = \frac{6b+b}{2} = \frac{7b}{2}$ *or* $\frac{7}{2}b$ $\frac{3b}{8} + \frac{b}{4} = \frac{3b+2b}{8} = \frac{5b}{8}$ *or* $\frac{5}{8}b$ Distributive law with a constant $5(2a - 3) = 10a - 15$ $5(2a - 3 + 5c) = 10a - 15 + 25c$ Include examples from other strands, for example area and perimeter of a rectangle  Find an expression for the area of the rectangle. Find an expression for the perimeter of the rectangle.	Factorising By identifying the highest common factor $4c - 8 = 4(c - 2)$ $3ab + 2a = a(3b + 2)$ $3a^2 - 12a = 3a(a - 4)$ Ensure learners recognise the relationship between expanding and factorising. Include examples from other strands, for example area and perimeter of a rectangle e.g. An expression for the area of this rectangle is $2b^2 + 14b$.  Find an expression for the width of the rectangle.	(perfect squares) Laws of indices $x^5 \times x^4 = x^{5+4} = x^9$ $y^8 \div y^3 = y^{8-3} = y^5$ $z^3 \times 3z = 1z^3 \times 3z^1 = 3z^{3+1} = 3z^4$ $4z^5 \div 2z^2 = 2z^{5-2} = 2z^3$ $(a^3)^2 = a^{3\times 2} = a^6$ $3n^2 - 2n^2 = n^2$ Algebraic fractions Ensure learners understand that 'cancelling' does not mean 'crossing out what is similar'. e.g. The first example is incorrect $\frac{6x+\cancel{3}}{\cancel{3}} = 6x,$ because learners need to factorise first as shown here: $\frac{6x+3}{3} = \frac{\cancel{3}(2x+1)}{\cancel{3}} = 2x + 1$
7Ae.04 Understand that a situation can be represented either in words or as an algebraic expression, and move between the two representations (linear with integer coefficients).	**8Ae.04** Understand that a situation can be represented either in words or as an algebraic expression, and move between the two representations (linear with integer or fractional coefficients).	**9Ae.03** Understand that a situation can be represented either in words or as an algebraic expression, and move between the two representations (including squares, cubes and roots).
Use examples with one or two steps and no brackets e.g. 7 more than double a number can be represented by $2n + 7$. Encourage learners to evaluate them by substituting a given value for each variable.	Use examples with more than two steps, more than one variable and with brackets e.g. a triangle has three sides, the shortest side has length x, the second side is twice as long as the shortest side and the longest side is 3 units longer than the second side. The expression for the perimeter of the triangle is $x + 2x + 2x + 3 = 5x + 3$	Use examples with more than one variable and with brackets e.g. I think of a number, n, multiply it by 2, square and then subtract 3. My number is represented by the expression $(2n)^2 - 3$

e.g. In the example above, if my number is 3. $2 \times 3 + 7 = 6 + 7 = 13$ (the double of my number is 6. 7 more than 6 is 13) If my number is 5. $2 \times 5 + 7 = 10 + 7 = 17$ (the double of my number is 10. 7 more than 10 is 17) Use examples where learners have to do the opposite: write an algebraic expression in words e.g. one step: $2x$ = the double of a number two step: $3n + 8$ = I have a number. I triple it and add eight.	x represents the shortest side, $2x$ represents the second side and $2x + 3$ represents the longest side. The area of a rectangle with side lengths $3a$ and $\frac{1}{2}b$ is represented by $$3a \times \frac{1}{2}b = \frac{3}{2}ab$$ Angelique has m marbles, Eva has 5 more marbles than Angelique, Carlos has twice as many marbles as Eva. Eva has $m + 5$ Carlos has $2(m + 5) = 2m + 10$ marbles. Encourage learners to evaluate them by substituting a given value for each variable. e.g. If Angelique has 5 marbles ($m = 5$), Eva will have $m + 5 = 5 + 5 = 10$ and Carlos $2 \times 5 + 10 = 10 + 10 = 20$. (20 is twice as many as 10, and 10 is 5 more than 5) If Angelique has 7 marbles, Eva will have 12 and Carlos will have 24. (24 is twice as many as 12, and 12 is five more than 7)	A rectangle has width x. The length of the rectangle is 3 more than twice the width. $Width = x$ $Length = 2x + 3$ The area of this triangle is represented by $Area = width \times length$ $x(2x + 3) = 2x^2 + 3x$ Encourage learners to evaluate them by substituting a given value for the variables. e.g. In the example above, if the width is 3, the length is $2 \times 3 + 3 = 6 + 3 = 9$ (6 is twice the width, and 9 is 3 more than 6). The area, therefore, needs to be $3 \times 9 = 27$. $2x^2 + 3x = 2 \times 3^2 + 3 \times 3 = 2 \times 9 + 9 = 18 + 9 = 27$, which shows that the expression for the area is correct. A square has sides length x. Using Pythagoras, learners are able to represent its diagonal as $\sqrt{2}x$.
7Ae.05 Understand that a situation can be represented either in words or as a formula (single operation), and move between the two representations.	**8Ae.05** Understand that a situation can be represented either in words or as a formula (mixed operations), and manipulate using knowledge of inverse operations to change the subject of a formula.	**9Ae.04** Understand that a situation can be represented either in words or as a formula (including squares and cubes), and manipulate using knowledge of inverse operations to change the subject of a formula.
Use examples where learners have to derive the formulae such as to convert hours to minutes ($m = 60h$); to convert centimetres to metres ($m = \frac{c}{100}$) Ensure learners use knowledge of inverse operations to manipulate formulae.	Use examples where learners have to derive the formulae such as to convert degrees Celsius (°C) to degrees Fahrenheit (°F) ($F = C \times 1.8 + 32$); to find the sum of the interior angles of a polygon of n sides ($s = 180(n - 2)$); cost of a taxi journey based on a $5 fixed charge and $2 per kilometre ($c = 2k + 5$); area of a trapezium ($A = \frac{a+b}{2}h$), which is also covered in Geometry and Measure; Ensure learners understand the fixed and variable components of formulae, and that they use knowledge of	e.g. Euler's formula for the relationship between the number of faces, vertices and edges in a polyhedron covered in Geometry and measure ($F + V - E = 2$); Ensure learners use knowledge of inverse operations to manipulate formulae. e.g. Rearrange $R = \frac{7c+3}{5}$ to make c the subject of the formula $$R = \frac{7c + 3}{5}$$ $5R = 7c + 3$

	inverse operations to manipulate formulae (rearranging up to two steps). e.g. Rearrange $R = 7c + 3$ to make c the subject of the formula $R = 7c + 3$ $R - 3 = 7c$ $\frac{R-3}{7} = c$ $c = \frac{R-3}{7}$	$5R - 3 = 7c$ $\frac{5R-3}{7} = c$ $c = \frac{5R-3}{7}$ e.g. A rectangle has width x, length y, and diagonal d. Using Pythagoras, learners are able to represent its diagonal d $d^2 = x^2 + y^2$. $d = \sqrt{x^2 + y^2}$.
7Ae.06 Understand that a situation can be represented either in words or as an equation. Move between the two representations and solve the equation (integer coefficients, unknown on one side).	**8Ae.06** Understand that a situation can be represented either in words or as an equation. Move between the two representations and solve the equation (integer or fractional coefficients, unknown on either or both sides).	**9Ae.05** Understand that a situation can be represented either in words or as an equation. Move between the two representations and solve the equation (including those with an unknown in the denominator).
e.g. $2x = 8$, $3x + 5 = 14$, $9 - 2x = 7$, $11 = 3x + 5$ I think of a number, double it and add 5, my answer is 13. The equation that represents this statement is $2n + 5 = 13$ Ensure learners understand that "taking or moving to the other side" means performing inverse operations. e.g. for the equation $x - 3 = 11$, we add the inverse to each expression. $x - 3 = 11$ $+3 = +3$ $x + 0 = 14$	e.g. $4x + 5 = 2x + 11$ $2x = 4(x - 3)$ $4(2x + 1) - 2(x - 5) = 2$ $\frac{3}{4}x = 16$ $3\left(\frac{7}{8}x + 2\right) = -18x$ To work out the sizes of the angles in the isosceles triangle below  $2x + 10$ is equal to $3x - 6$ due to the characteristics of an isosceles triangle.	e.g. $\frac{15}{z} = 5$ $15 = 5z$ $3 = z$ e.g. $\frac{2}{x+1} = 16$ There are x students in room A. There are 3 more students in room B than in room A. I share 100 sweets equally between all the students in room B and they receive 20 each. This problem can be represented by the equation $\frac{100}{x+3} = 20$, where x is the number of students in room A.

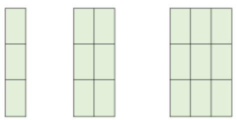
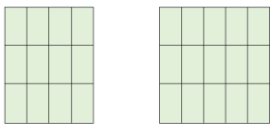
That gives us $-3 + 3 = 0$ on the left and $11 + 3 = 14$ on the right, so the solution of the equation is $x = 14$	Therefore, $2x + 10 = 3x - 6$ $2x - 3x = -6 - 10$ $-x = -16$ $x = 16$ Substituting: $2 \times 16 + 10 = 32 + 10 = 42$ $180 - (42 + 42) = 180 - 84 = 96$ The angles are $42°, 42°, 96°$	Solving gives $x = 2$, so there are 2 students in room A and 5 students in room B
		9Ae.06 Understand that the solution of simultaneous linear equations: - is the pair of values that satisfy both equations - can be found algebraically (eliminating one variable) - can be found graphically (point of intersection).
		Use examples with a pair of equations, including with decimal answers (approximate solutions). e.g. The graph shows the lines $y = x + 1$ and $x + y = 3$ 

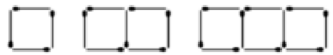
		The point of intersection is $(1, 2)$ and therefore the solution of the simultaneous equations is $x = 1, y = 2$. The same solution to the set of simultaneous linear equations $y = x + 1$ $x + y = 3$ can also be found algebraically. Using substitution: $x + y = 3$ $x + x + 1 = 3$ (substituting x+1 for y) $2x + 1 = 3$ $2x = 2$ $x = 1$ $y = x + 1$ $y = 1 + 1 = 2$ A pair of simultaneous equations can also be solved using elimination: $6x + 5y = 38$ $2x + 5y = 26$ Subtract to reach $4x = 12$ $x = 3$ $2x + 5y = 26$ $6 + 5y = 26$ $5y = 20$ $y = 4$

7Ae.07 Understand that letters can represent an open interval (one term).	**8Ae.07** Understand that letters can represent open and closed intervals (two terms).	**9Ae.07** Understand that a situation can be represented either in words or as an inequality. Move between the two representations and solve linear inequalities.
Ensure learners understand that $x < 3$ is represented as  which means that the 3 does not belong to the interval and that the interval goes to negative infinity (x can assume any number below 3) This uses their knowledge of rational numbers and builds to solve inequalities in later stages.	e.g. $x \leq 2$ (x can assume any number below 2, including 2) $-2 < x \leq 2$ (x can assume any number between -2 and 2, excluding -2 and including 2) Use examples that require learners to use equivalence of intervals $x > 3$ is equivalent to $2x > 6$ If $x > 3$, $x - 4 > 3 - 4$ $x - 4 > -1$	e.g. The inequality $3x + 1 > -5$ is true for any $x > -2$  The inequality $-3 < x + 1 \leq 2$ is true when $-3 - 1 < x + 1 - 1 \leq 2 - 1$ $-4 < x \leq 1$ Its solution can be represented as  Ensure learners understand the difference between equations (solution is a point) and inequalities (solution is an interval). Use examples with negative coefficients, but not focusing only on the rule of flipping the sign when multiplying by -1. e.g. $3x + 1 > 4x - 2$ $3x - 4x > -2 - 1$ $-x > -3$ $x < 3$ Encourage learners to use numbers to check the equivalency.

		For $x = 2 (< 3)$ $3 \times 2 + 1 > 4 \times 2 - 2$ $6 + 1 > 8 - 2$ $7 > 6$ (true) For $x = 3$ $3 \times 3 + 1 > 4 \times 3 - 2$ $9 + 1 > 12 - 2$ $10 > 10$ (false)

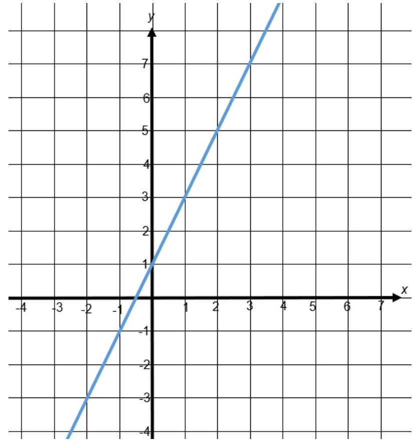
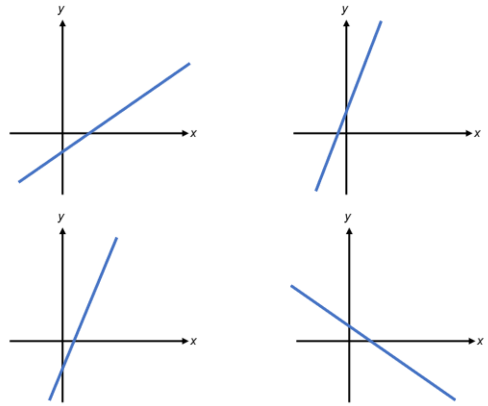
Algebra
Sequences, functions and graphs

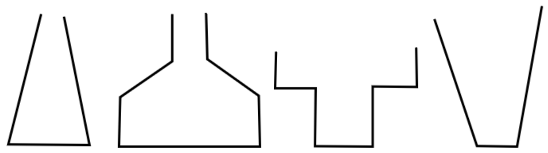
Stage 7	Stage 8	Stage 9
7As.01 Understand term-to-term rules, and generate sequences from numerical and spatial patterns (linear and integers).	**8As.01** Understand term-to-term rules, and generate sequences from numerical and spatial patterns (including fractions).	**9As.01** Generate linear and quadratic sequences from numerical patterns and from a given term-to-term rule (any indices).
e.g. Continue the sequence -4, -7, -10, -13, …. The term-to-term rule is subtract 3 so the next term is -16. A sequence has first term 3 and term-to-term rule add 5. The next terms in the sequence can be found by applying the term-to-term rule to the first term $3 + 5 = 8$ $8 + 5 = 13$ $13 + 5 = 18$ The first four terms are: 3, 8, 13, 18. The next patterns in the sequence  are 	e.g. A sequence has first term ½ and the term-to-term rule is add 1 ½. The first four terms in the sequence are ½, 2, 3½, 5. A sequence has first term 2 and the term-to-term rule is multiply by 3 and add 1. The next terms in the sequence can be found by applying the term-to-term rule to the first term $2 \times 3 + 1 = 6 + 1 = 7$ $7 \times 3 + 1 = 21 + 1 = 22$ $22 \times 3 + 1 = 66 + 1 = 67$ $67 \times 3 + 1 = 201 + 1 = 202$ The first five terms are: 2, 7, 22, 67, 202.	e.g. A sequence has first term 2 and term-to-term rule square and subtract 1. The next terms in the sequence can be found by applying the term-to-term rule to the first term $2^2 - 1 = 4 - 1 = 3$ $3^2 - 1 = 9 - 1 = 8$ $8^2 - 1 = 64 - 1 = 63$ The first four terms are: 2, 3, 8, 63.
7As.02 Understand and describe nth term rules algebraically (in the form $n \pm a$, $a \times n$ where a is a whole number).	**8As.02** Understand and describe nth term rules algebraically (in the form $n \pm a$, $a \times n$, or $an \pm b$, where a and b are positive or negative integers or fractions).	**9As.02** Understand and describe nth term rules algebraically (in the form $an \pm b$, where a and b are positive or negative integers or fractions, and in the form $\frac{n}{a}$, n^2, n^3 or $n^2 \pm a$, where a is a whole number).
If the nth term (position-to-term rule) for a sequence is given as $n-3$. The 12th term can be found $12 - 3 = 9$ Ensure learners understand how to find the nth term rule for an easy linear sequence (in the form $n \pm a$, $a \times n$)	A sequence has nth term $20 - \frac{1}{2}n$. The 6th term can be found by substituting n with 6: $20 - \frac{1}{2} \times 6 = 20 - 3 = 17$.	Examples can include generating sequences of the form $n^2 \pm a$. A sequence has nth term $n^2 + 1$. The 6th term of this sequence can be found by replacing n with 6: $6^2 + 1 = 36 + 1 = 37$.

e.g In the sequence $2, 4, 6, 8, \ldots$ the nth term rule (position-to-term rule) is 2n $2 \times 1 = 2$ $2 \times 2 = 4$ $2 \times 3 = 6$ $2 \times 4 = 8$ $\vdots$ $2 \times n = 2n$	Ensure learners understand how to find the nth term rule for a given linear sequence e.g. The nth term for the sequence: $1.4, 2.6, 3.8, 5.0 \ldots$ is $1.2n + 0.2$ In the sequence  the position-to-term rule (nth term rule) for the number of matchsticks $3n + 1$. Learners should be able to identify if a number is part of a sequence given by an nth term rule. e.g. The nth term of a sequence is given by the rule 3n + 5. Could the number 39 be a term in the sequence? This could be done by generating terms of the sequence to see if 39 is part of the sequence, or by solving the equation 3n + 5 = 39 to see if this gives an integer value.	Learners can explore patterns which occur in the difference between terms and the difference between these differences (second difference). e.g. If the nth term of a sequence is given by the rule $n^2 + 4$, the first 5 terms of the sequence will be: 5, 8, 13, 20, 29. The difference between each pair of terms is 3, 5, 7, 9 and the difference between each of these is 2, 2, 2 Sequences with nth term of the form $n^2 + a$ can be identified by comparing with the square number sequence (n^2) e.g. In the sequence 3, 6, 11, 18, 27, … each term is two more than its corresponding term in the square number sequence and so the nth term for this sequence is given by the rule $n^2 + 2$ Ensure learners understand how to find the nth term rule for a given sequence In the sequence $\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}$ ... the nth term is $\frac{n}{3}$ Ensure learners recognise the sequence of square and cube numbers and can give their nth term rule e.g. For the sequence 1, 8, 27, 64,…, the nth term rule is n^3
7As.03 Understand that a function is a relationship where each input has a single output. Generate outputs from a given function and identify inputs from a given output by considering inverse operations (linear and integers).	**8As.03** Understand that a function is a relationship where each input has a single output. Generate outputs from a given function and identify inputs from a given output by considering inverse operations (including fractions).	**9As.03** Understand that a function is a relationship where each input has a single output. Generate outputs from a given function and identify inputs from a given output by considering inverse operations (including indices).
Numerical and diagrammatic representations of functions and algebraic representations of functions of the form $y = x + c$ or $y = mx$. Functions machines (including inverse), mapping diagrams, tables, etc. can be used to help learner understanding. Ensure learners can understand and use different representations of one-step functions.	Numerical and diagrammatic representations of functions and algebraic representations of functions of the form $y = mx + c$. Functions machines (including inverse), mapping diagrams, tables, etc. can be used to help learner understanding.	Numerical, algebraic and diagrammatic representations. Functions machines (including inverse), mapping diagrams, tables, etc. can be used to help learner understanding. e.g. Create a function machine for $y = 3x^2$ and consider different inputs such as 1, -1, $\frac{1}{3}$ etc.

e.g. the function $y = x + 2$ can be represented by the function machine INPUT → $\boxed{+2}$ → OUTPUT an input of 3 would give an output of 5 an input of -1 would give an output of 1 e.g. for the function $y = x + 2$ the inverse function machine could be represented as INPUT ← $\boxed{-2}$ ← OUTPUT an output of 19 would give an input of 17 an output of 0 would give an input of -2	e.g. the function $y = 3x - 2$ can be represented by the function machine INPUT → $\boxed{\times 3}$ → $\boxed{-2}$ → OUTPUT an input of $\frac{1}{2}$ would give an output of $-\frac{1}{2}$ e.g. for the function $y = 3x - 2$ the inverse function machine could be represented as INPUT ← $\boxed{\div 3}$ ← $\boxed{+2}$ ← OUTPUT This inverse function machine can be used to find the input when the output is 2.5 $2.5 + 2 = 4.5$ $4.5 \div 3 = 1.5$ So the input was 1.5	e.g. For the function $y = 3x^2$ if the output is 12, the possible inputs are 2 and -2
7As.04 Understand that a situation can be represented either in words or as a linear function in two variables (of the form $y = x + c$ or $y = mx$), and move between the two representations.	**8As.04** Understand that a situation can be represented either in words or as a linear function in two variables (of the form $y = mx + c$), and move between the two representations.	**9As.04** Understand that a situation can be represented either in words or as a linear function in two variables (of the form $y = mx + c$ or $ax + by = c$), and move between the two representations.
Include examples with functions arising from real-life problems e.g. currency conversion If 1 Thai Baht (x) is exchanged for 3 Japanese Yen (y) then this can be represented by the function $y = 3x$ Use examples where the variables are letters other than x and y. e.g. c = 2k	Include examples with functions arising from real-life problems e.g. cost of hiring a cement mixer with a $25 fixed cost and additional charge of $10 per day can be represented by the function $y = 10x + 25$ where y is the total cost and x is the number of days. Use examples where the variables are letters other than x and y. e.g. c = 2k + 5	Include examples with functions arising from real-life problems e.g. large tables seat 10 people and small tables seat 5 people, 200 people need to be seated. This situation can be represented by the function $10x + 5y = 200$ where x is the number of large tables and y is the number of small tables Use examples where the variables are letters other than x and y. e.g. 2c + 5k = 10

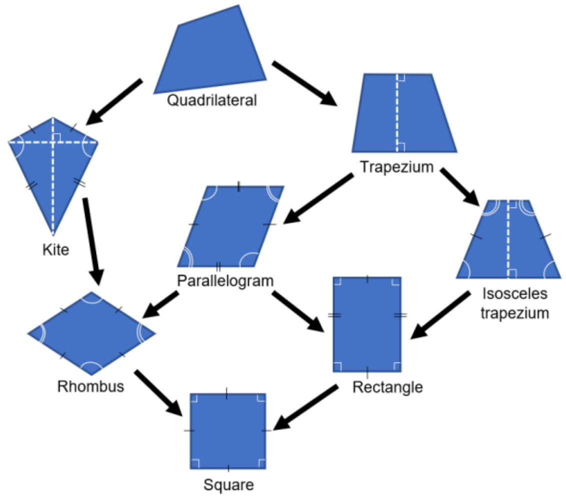
<table>
<tr>
<td>7As.05 Use knowledge of coordinate pairs to construct tables of values and plot the graphs of linear functions, where y is given explicitly in terms of x ($y = x + c$ or $y = mx$).</td>
<td>8As.05 Use knowledge of coordinate pairs to construct tables of values and plot the graphs of linear functions, where y is given explicitly in terms of x ($y = mx + c$).</td>
<td>9As.05 Use knowledge of coordinate pairs to construct tables of values and plot the graphs of linear functions, including where y is given implicitly in terms of x ($ax + by = c$), and quadratic functions of the form $y = x^2 \pm a$.</td>
</tr>
<tr>
<td>Numerical and graphic representations of functions
e.g. a completed table of values for $y = x + 2$
<table>
<tr><td>x</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>y</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
</table>
gives us the coordinate pairs (-1,1), (0,2), (1,3), (2,4), (3,5) to aid in plotting the graph of $y = x + 2$
e.g. $y = x + 2$
If the x coordinate is 12, the y coordinate is 14.
So (12,14) lies on the line
Include examples with functions arising from real-life problems and learners have to draw and interpret their graphs.
e.g. currency conversion
e.g. A document with 110 pages takes 5 minutes to print. Draw a straight line graph to show how the time (t minutes) to print a document varies with the number of pages (x).</td>
<td>Numerical and graphic representations of functions
e.g. a completed table of values for $y = 3x - 2$
<table>
<tr><td>x</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>y</td><td>-5</td><td>-2</td><td>1</td><td>4</td><td>7</td></tr>
</table>
gives us the coordinate pairs (-1,-5), (0,-2), (1,1), (2,4), (3,7) to aid in plotting the graph of $y = 3x - 2$
e.g. $y = 4x - 8$
If the y coordinate is 0, the x coordinate is 2.
So (2, 0) lies on the line and this is where it crosses the x axis.
If the x coordinate is 0, the y coordinate is -8.
So (0, -8) lies on the line and this is where it crosses the y axis.
Include examples with functions arising from real-life problems and learners have to draw and interpret their graphs.
e.g. cost of hiring a cement mixer with a $25 fixed cost and additional charge of $10 per day.</td>
<td>Numerical and graphic representations of functions
e.g. a completed table of values for $10x + 5y = 200$
<table>
<tr><td>x</td><td>0</td><td>5</td><td>10</td><td>15</td><td>20</td></tr>
<tr><td>y</td><td>40</td><td>30</td><td>20</td><td>10</td><td>0</td></tr>
</table>
e.g. a completed table of values for $y = x^2 + 5$
<table>
<tr><td>x</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>y</td><td>6</td><td>5</td><td>6</td><td>9</td><td>14</td></tr>
</table>
Include examples where learners have to draw and interpret their graphs.
e.g. $4x + 3y = 12$
If the y coordinate is 0, the x coordinate is 3.
So (3, 0) lies on the line and this is where it crosses the x axis.
If the x coordinate is 0, the y coordinate is 4.
So (0, 4) lies on the line and this is where it crosses the y axis.</td>
</tr>
<tr>
<td>7As.06 Recognise straight-line graphs parallel to the x- or y-axis.</td>
<td>8As.06 Recognise that equations of the form $y = mx + c$ correspond to straight-line graphs, where m is the gradient and c is the y-intercept (integer values of m).</td>
<td>9As.06 Understand that straight-line graphs can be represented by equations. Find the equation in the form $y = mx + c$ or where y is given implicitly in terms of x (fractional, positive and negative gradients).</td>
</tr>
<tr>
<td>Deduce that graphs of the form $x = a$ will be parallel to the y-axis and graphs of the form $y = b$ will be parallel to the x-axis</td>
<td>Ensure learners recognise the graph of the particular case $y = x$.</td>
<td>Include examples which require learners to find the gradient and y-intercept from a graph and deduce the equation of the graph in the form $y = mx + c$</td>
</tr>
</table>

e.g. recognise and label the graphs of $$x = 5$$ For this graph the x coordinate of each point on the line will always be 5, so the coordinates for this line will be (5,0), (5,1), (5,2), (5,3) and so on. $y = -3$ For this graph the y coordinate of each point on the line will always be -3, so the coordinates for this line will be (0,-3), (1,-3), (2,-3), (3,-3) and so on.	Learners should be able to plot and identify connections between families of graphs. e.g. after plotting the graphs of $y = 2x + 3$ $y = 2x + 1$ $y = 2x - 2$ They should be able to identify that the three lines are parallel to each other and all have the same gradient. They should be able to identify that each graph has a different y-intercept and that the y-intercept for each graph is the same as the constant in the equation of the graph. Learners should also be able to plot graphs with the same y-intercept and notice what effect the value of the x coefficient has on the gradient of the line. Include examples with functions arising from real-life problems and learners have to interpret the features of the graph e.g. cost of hiring a cement mixer with a \$25 fixed cost and additional charge of \$10 per day. $$y = 25 + 10x$$ y is the total cost $c = 25$ (y-intercept) $m = 10$ (gradient) x is the number of days	e.g. What is the gradient? What is the y-intercept? What is the equation of this graph?  e.g. Tick the graph of $y = 2x + 1$  Include examples which require learners to rearrange equations to make y the subject and to identify the gradient and the y-intercept. e.g. the equation $3y + 2x - 7 = 26$ can be rearranged to $y = -\frac{2}{3}x + 11$ Both equations represent the straight line graph with gradient $-\frac{2}{3}$ and y-intercept (0,11)

7As.07 Read and interpret graphs related to rates of change. Explain why they have a specific shape.	**8As.07** Read and interpret graphs with more than one component. Explain why they have a specific shape and the significance of intersections of the graphs.	**9As.07** Read, draw and interpret graphs and use compound measures to compare graphs.
e.g. line graphs (with a single line only), travel graphs (distance-time), temperature graphs, graph showing depth of water in a container over time when the container has a variety of shapes and liquid is added at a constant rate:  Ensure learners understand: • rate measures the change of a quantity in relation to time, and therefore the faster something changes the higher the rate • positive, zero and negative gradients • speed is a rate and represents a change in distance in relation to time	e.g. travel graphs (distance-time) with more than one person; a graph for phone bill against number of minutes used for two different contracts. Learners should not calculate further values/compound values etc.	This includes calculating further values or compound values from the graph e.g. Calculating the gradient to find a rate of change Currency exchange rate Price per unit Speed from a distance time graph at various points

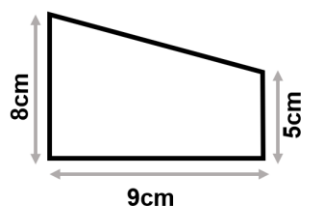
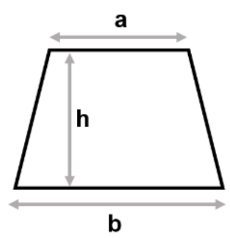
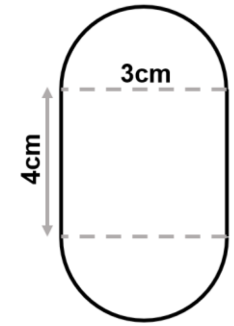
Geometry and Measure

Geometrical reasoning, shapes and measurements

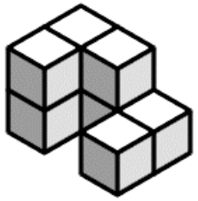
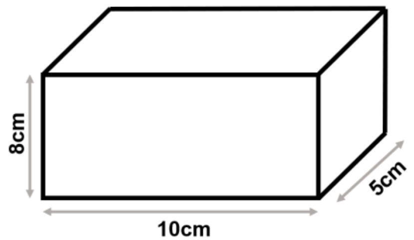
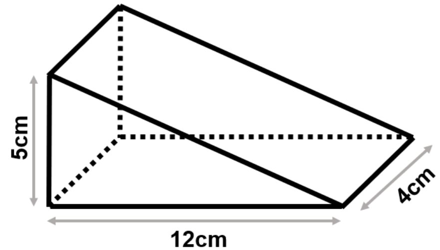
Stage 7	Stage 8	Stage 9
7Gg.01 Identify, describe and sketch regular polygons, including reference to sides, angles and symmetrical properties.	**8Gg.01** Identify and describe the hierarchy of quadrilaterals.	
e.g. understand the line symmetry and rotational symmetry properties of regular polygons	 The hierarchy includes seven quadrilaterals: kite, parallelogram, rhombus, rectangle, square, trapezium, isosceles trapezium. Here a trapezium is defined as a 2D shape with at least one pair of parallel sides. Give learners the opportunity to investigate the necessary and sufficient properties to define each specific type of quadrilateral. Ensure learners understand the hierarchy. e.g. All squares are rectangles, all rectangles are parallelograms etc.	

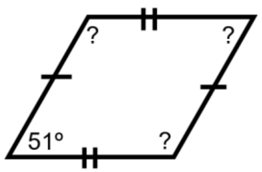
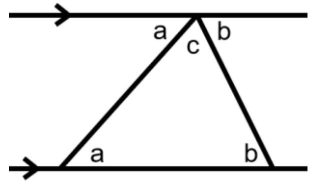
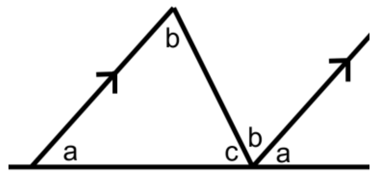
<table>
<tr><td>7Gg.02 Understand that if two 2D shapes are congruent, corresponding sides and angles are equal.</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>7Gg.03 Know the parts of a circle:
- centre
- radius
- diameter
- circumference
- chord
- tangent.</td><td>8Gg.02 Understand π as the ratio between a circumference and a diameter. Know and use the formula for the circumference of a circle.</td><td>9Gg.01 Know and use the formulae for the area and circumference of a circle.</td></tr>
<tr><td>Recognise and label the parts of a circle.
Ensure learners understand that any length inside the circle that does not pass through the centre is not a diameter, it is a chord.
Know that the tangent and radius are perpendicular to each other.</td><td>Learners could explore the ratio between the circumference and diameter of circular objects by measuring.
Learners should understand that π can be approximated as 3.142, $\frac{22}{7}$ and that there is a value of π stored on a scientific calculator.

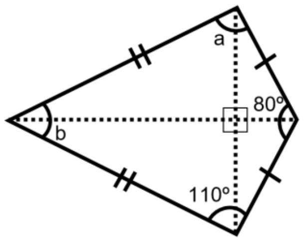
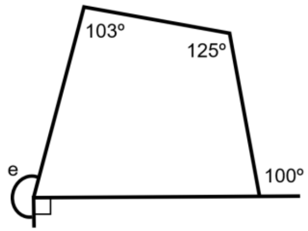
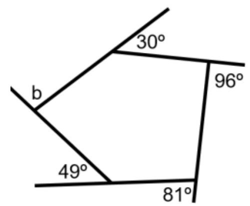
Circumference
C = $2\pi r$ or $\pi \mathrm{d}$.
e.g. The radius of a circle is 3.2cm, find the circumference.
Radius = 3.2cm
Diameter = 6.4cm
C = 6.4 x π = 20.11cm to 2 d.p.
e.g. The circumference of a circle is 20.8cm, find the diameter of the circle.
d = 20.8 ÷ π =
6.62cm to 2 d.p.</td><td>Area
$$A = \pi r^2$$
e.g. The diameter of a circle is 10.8cm, find its area.
d = 10.8cm
r = 5.4cm
A = π x 5.4^2 =
91.69cm^2 to 2d.p.
Include examples of finding the area and perimeter of semi-circles.</td></tr>
</table>

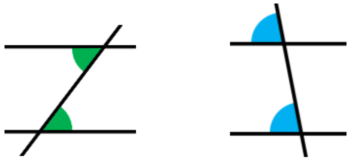
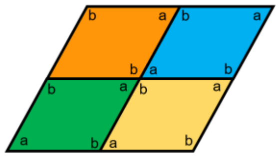
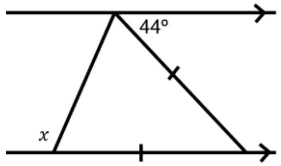
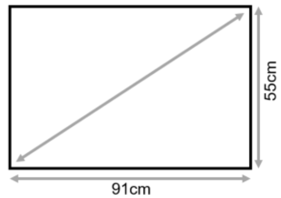
7Gg.04 Understand the relationships and convert between metric units of area, including hectares (ha), square metres (m^2), square centimetres (cm^2) and square millimetres (mm^2).	**8Gg.03** Know that distances can be measured in miles or kilometres, and that a kilometre is approximately $\frac{5}{8}$ of a mile or a mile is 1.6 kilometres.	**9Gg.02** Know and recognise very small or very large units of length, capacity and mass.
e.g. The floor of a room measures 3m by 2.4m. Some tiles measure 20cm by 20cm. Work out how many tiles are needed to cover the floor of the room. Learners should understand that one square centimetre is 10mm x 10mm and therefore $1cm^2 = 100mm^2$ $1ha = 10\ 000m^2$ $1m^2 = 10{,}000\ cm^2$ $1km^2 = 1{,}000{,}000m^2$ e.g. recognise that 12450 cm^2 is 1.245 m^2	e.g. Which is longer, 85 km or 52 miles?	1 light year (ly - 9 460 730 472 580 800 metres), 1 micrometre (μm - one millionth of a metre), 1 nanometre (nm-one billionth of a metre), milligram (mg - one thousandth of a gram), microgram (μg - one millionth of a gram), tonne (t - 1000kg). microlitre (μl), megabyte (MB), gigabyte (GB), terabyte (TB).
7Gg.05 Derive and know the formula for the area of a triangle. Use the formula to calculate the area of triangles and compound shapes made from rectangles and triangles.	**8Gg.04** Use knowledge of rectangles, squares and triangles to derive the formulae for the area of parallelograms and trapezia. Use the formulae to calculate the area of parallelograms and trapezia.	**9Gg.03** Estimate and calculate areas of compound 2D shapes made from rectangles, triangles and circles.
e.g. thinking of a triangle as half a rectangle e.g. use what you know about triangles and rectangles to find the area of this shape. 	Area of a parallelogram = base x perpendicular height Area of a trapezium $A = \frac{1}{2}(a + b)h$ 	e.g. find the area of this shape. 

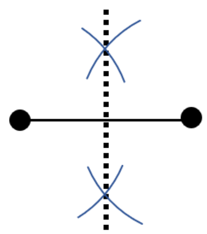
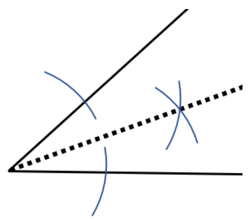
7Gg.06 Identify and describe the combination of properties that determine a specific 3D shape.	**8Gg.05** Understand and use Euler's formula to connect number of vertices, faces and edges of 3D shapes.	
Learners should understand that a 'face' is, by definition, flat. If curved, it is described as a 'curved surface'. e.g. a 3D shape has 1 curved surface, no edges and no vertices, so it must be a sphere e.g. A pyramid will always have the same number of faces as vertices. Include less familiar properties such as: • All pyramids have an even number of edges. • The number of edges of a prism is a multiple of 3	Euler's formula is V + F – E = 2 Ensure learners know that Euler's formula only applies to polyhedra.	
7Gg.07 Derive and use a formula for the volume of a cube or cuboid. Use the formula to calculate the volume of compound shapes made from cuboids, in cubic metres (m^3), cubic centimetres (cm^3) and cubic millimetres (mm^3).	**8Gg.06** Use knowledge of area and volume to derive the formula for the volume of a triangular prism. Use the formula to calculate the volume of triangular prisms.	**9Gg.04** Use knowledge of area and volume to derive the formula for the volume of prisms and cylinders. Use the formula to calculate the volume of prisms and cylinders.
Introduce many examples of volume so that learners can find the formula volume = length x width x height Include examples of working backwards to find a side length of a cuboid when given the volume and 2 dimensions. e.g. The volume of a cuboid is $120cm^3$. The height of the cuboid is 10cm and the length is 4cm. Find the width. Learners should be aware of the relationship between cubic centimetres and litres.	Use properties of rectangles to calculate area of a triangle. Volume of triangular prism = area of triangular cross section x length	Use properties of circles to find the area of a circle. Volume of cylinder = area of circular cross section x length Generalise that for any prism: Volume = area of cross section x length
7Gg.08 Visualise and represent front, side and top view of 3D shapes.	**8Gg.07** Represent front, side and top view of 3D shapes to scale.	
e.g. Draw the front, side and top view of this 3D shape	Use knowledge of ratio and proportion to create scale drawings of 3D shapes.	

 Front, side and top view can also be referred to as plans and elevations.	Ensure learners understand that a 2D representation of a 3D shape can be a scale drawing or not. When drawing blueprints, for example, the proportions should be kept e.g. 5m should be equivalent to 5cm for all parts.	
7Gg.09 Use knowledge of area, and properties of cubes and cuboids to calculate their surface area.	**8Gg.08** Use knowledge of area, and properties of cubes, cuboids, triangular prisms and pyramids to calculate their surface area.	**9Gg.05** Use knowledge of area, and properties of cubes, cuboids, triangular prisms, pyramids and cylinders to calculate their surface area.
Include reference to the formula for the surface area of a cube ($6s^2$) Learners should be able to use the net of a cuboid to determine the rectangles that make up the faces of the cuboid and work out its surface area. e.g. Find the surface area of this cuboid  2 x (8 x 10) = 160 2 x (5 x 8) = 80 2 x (10 x 5) = 100 160 + 80 + 100 = 340cm^2	Learners should be able to use the nets of shapes to help them to determine the areas of each of the faces of the shape.	Connect right-angled prisms to Pythagoras' theorem. e.g. Find the surface area of this triangular prism.  (Use Pythagoras theorem to find the missing length on the sloping side)
7Gg.10 Identify reflective symmetry and order of rotational symmetry of 2D shapes and patterns.	**8Gg.09** Understand that the number of sides of a regular polygon is equal to the number of lines of symmetry and the order of rotation.	**9Gg.06** Identify reflective symmetry in 3D shapes.

A picture of star showing three mirror lines or fold lines and order five rotational symmetry  e.g. (isosceles) trapezium, kite, parallelogram, rhombus, isosceles triangle, equilateral triangle, scalene triangle, shapes that have curves Some shapes can have rotational but not reflective symmetry.	e.g. a regular pentagon has 5 lines of symmetry and rotational symmetry order 5	e.g. find the number of planes of symmetry of a hexagonal prism with a regular hexagon as cross-section Ensure learners understand that in two dimensions, a shape is divided into 2 congruent parts by a line of symmetry. In three dimensions, a plane of symmetry divides a solid into two congruent parts.
7Gg.11 Derive the property that the sum of the angles in a quadrilateral is 360°, and use this to calculate missing angles.	**8Gg.10** Derive and use the fact that the exterior angle of a triangle is equal to the sum of the two interior opposite angles.	**9Gg.07** Derive and use the formula for the sum of the interior angles of any polygon.
e.g. draw the diagonal of a quadrilateral to form two triangles and use the angle sum of these to show that the angle sum of a quadrilateral is 2 x 180º. Include examples where learners have to apply their knowledge of the properties of quadrilaterals including reference to parallel sides, right angles, rotational and reflective symmetry. e.g.  Learners know that a parallelogram has rotation symmetry order 2. So the top right angle is also 51°. The remaining two angles are also equal: 360-51-51=258 258÷ 2 =129°	Interior angle sum of a triangle is 180°.  Exterior angle of a triangle is equal to the sum of the two interior opposite angles.  Learners should be combining angle facts about straight lines, triangles, around a point and quadrilaterals to find missing angles in diagrams.	e.g. a polygon with n sides can be split into n – 2 triangles each with an angle sum of 180°. The sum of the interior angles of a polygon is (n-2) x 180. For a regular polygon each of the interior angles will be equal. e.g. a regular pentagon has an interior angle sum of (5 - 2) x 180 = 540º 540 ÷ 5 = 108º Each interior angle is 108º Learners can use interior and exterior angles of regular polygons to consider why some tessellate and some not.

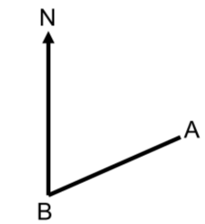
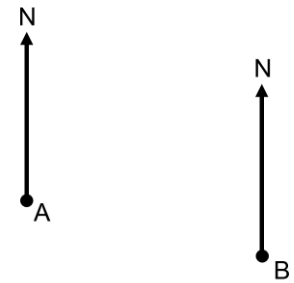
e.g.  Learners know there is a horizontal line of reflection so angle a=110° Then angles in a quadrilateral add to 360° 360- 110- 110- 80 = 60 So b=60°	e.g. Find the size of angle e 	
7Gg.12 Know that the sum of the angles around a point is 360º, and use this to calculate missing angles.		**9Gg.08** Know that the sum of the exterior angles of any polygon is 360°.
		The exterior angles of a polygon create a journey around the shape that totals 360º e.g.  Learners can use interior and exterior angles of regular polygons to consider why some tessellate and some not.
7Gg.13 Recognise the properties of angles on: - parallel lines and transversals - perpendicular lines - intersecting lines.	**8Gg.11** Recognise and describe the properties of angles on parallel and intersecting lines, using geometric vocabulary such as alternate, corresponding and vertically opposite.	**9Gg.09** Use properties of angles, parallel and intersecting lines, triangles and quadrilaterals to calculate missing angles.

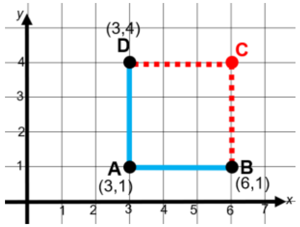
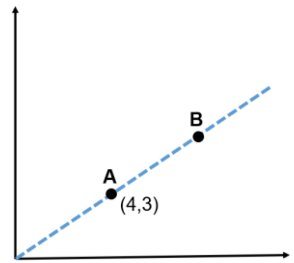
e.g. Find equal angles in a simple diagram with parallel lines and a transversal  Find equal angles formed where two lines intersect  Identify parallel lines and equal angles in tessellating parallelograms 	e.g. find equal angles in more complex diagrams, and explain why they are equal, with the correct geometrical language alternate, corresponding, vertically opposite	e.g. include examples involving triangle, quadrilaterals and parallel lines where a number of properties need to be applied together.  Calculate the size of the angle marked x.
		9Gg.10 Know and use Pythagoras' theorem.
		Learners should understand the relationship between the sides of a right-angled triangle and be able to use two sides of a right angled triangle to calculate the third side. Learners should be able to solve problems in a range of contexts e.g. Work out the length of the diagonal of the television screen 

7Gg.14 Draw parallel and perpendicular lines, and quadrilaterals.	**8Gg.12** Construct triangles, midpoint and perpendicular bisector of a line segment, and the bisector of an angle.	**9Gg.11** Construct 60º, 45º and 30º angles and regular polygons.
Using ruler, set square, a protractor or digital technology.	Using compass and ruler or digital technology. Construct triangles • given two sides and the included angle (SAS) • or two angles and the included side (ASA) • given three sides (SSS) • given a right angle, hypotenuse and one side (RHS) Use a pair of compasses to construct the perpendicular bisector of a line segment, knowing to show all construction lines  Use a pair of compasses to construct an angle bisector knowing to show all construction lines 	Using compass and ruler or digital technology. Includes Inscribed squares, equilateral triangles, and regular hexagons and octagons in a circle. e.g. construct a 45º angle by constructing a perpendicular bisector (90º) and then constructing an angle bisector.

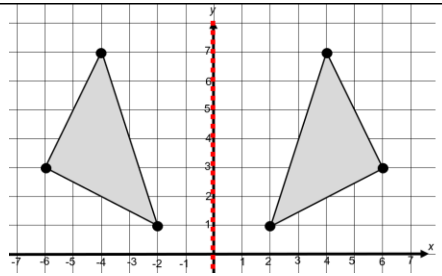
Geometry and Measure

Position and transformation

Stage 7	Stage 8	Stage 9
7Gp.01 Use knowledge of scaling to interpret maps and plans.	**8Gp.01** Understand and use bearings as a measure of direction.	**9Gp.01** Use knowledge of bearings and scaling to interpret position on maps and plans.
Learners should be able to convert from a map measurement to a real life distance and vice versa e.g. a map has a scale of 1:100 000 The distance between two towns on the map is 3.4cm What is the distance in real life? 3.4 x 100 000 = 340 000cm = 3400m = 3.4km	Bearings are angles measured clockwise from north (three digits). Include examples where learners need to measure a bearing using a protractor or draw a bearing using a protractor e.g. what is the bearing of A from B.  e.g. Draw a bearing of 035°	Ensure learners can use bearings to interpret all possible locations or use two bearings to find the exact location. e.g. The diagram shows the position of two mountains, A and B.  A third mountain, C, is on a bearing of 145° from A and on a bearing of 270° from B. Mark the position of C on the diagram. e.g. a tree is 40m from a house and is on a bearing of 056° from the house. Use the scale to mark the position of the tree on the map below…
7Gp.02 Use knowledge of 2D shapes and coordinates to find the distance between two coordinates that have the same x or y coordinate (without the aid of a grid).	**8Gp.02** Use knowledge of coordinates to find the midpoint of a line segment.	**9Gp.02** Use knowledge of coordinates to find points on a line segment.

 Ensure learners understand that (3, 1) and (6,1) belong to the same horizontal line. Similarly, ensure learners understand that for example, (1,3) and (1,6) belong to the same vertical line. This is a misconception as they see the x coordinates are the same and incorrectly state that the coordinates belong to the horizontal line. Learners should be able to identify the coordinate of C as (6,4) Learners should be able to solve similar problems involving squares and rectangles that are tilted, parallelograms, isosceles triangles etc.	Learners should be encouraged to find the midpoint of a line segment both geometrically by measuring and by calculation. Learners should be able to solve problems where the midpoint is given. e.g. Point M (4.9) is the midpoint of point A (2,1) and point B (?,?) Find the coordinates of point B	One third of the distance of the y axis and one third of the distance of the x axis Find the point that lies one third of the way along the line segment joining (0,0) and (6,12) e.g. The origin O, point A, and point B are equally spaced along the same line such that the distance OA is equal to the distance AB. A is the point (4, 3). What are the coordinates of point B? If the points continue along the line so that each subsequent point is labelled with the next letter of the alphabet, what would be the coordinates of point T? 
7Gp.03 Use knowledge of translation of 2D shapes to identify the corresponding points between the original and the translated image, without the use of a grid.	**8Gp.03** Translate points and 2D shapes using vectors, recognising that the image is congruent to the object after a translation.	**9Gp.03** Transform points and 2D shapes by combinations of reflections, translations and rotations.
e.g. The point A has coordinates (3,4). The point A is translated 2 left and 2 up. What are the new coordinates of the position of point A now?	Ensure learners understand the difference between a vector and coordinate, including vector notation. e.g. translating a shape using the vector $\begin{pmatrix}2\\5\end{pmatrix}$ means the shape moves right 2 and up 5 Ensure learners understand that if two 2D shapes are congruent, corresponding sides and angles are equal	e.g. Reflect the shape in the line x=3 and then reflect the new shape in the line y=5 e.g. Translate the shape by the vector $\begin{pmatrix}2\\5\end{pmatrix}$ and then rotate the new shape by 180° around the point (2,1)

	Understand that the image is congruent to the object after these transformations (they preserve length and angles) Ensure learners can identify the vector for a given translation of an object to its image.	
		9Gp.04 Identify and describe a transformation (reflections, translations, rotations and combinations of these) given an object and its image.
		Ensure learners understand what is needed to give a precise description of a reflection, translation or rotation. e.g. Reflection requires the equation of the mirror line, translation requires a vector, rotation requires the angle and direction of rotation (clockwise / anticlockwise) e.g. Reflect shape A in the line x=2 and label it B. Reflect shape B in the line y=1 and label it C. What single transformation would take shape A to shape C?
		9Gp.05 Recognise and explain that after any combination of reflections, translations and rotations the image is congruent to the object.
7Gp.04 Reflect 2D shapes on coordinate grids, in a given mirror line (x- or y-axis), recognising that the image is congruent to the object after a reflection.	**8Gp.04** Reflect 2D shapes and points in a given mirror line on or parallel to the x- or y-axis, or $y = \pm x$ on coordinate grids. Identify a reflection and its mirror line.	
	Includes identifying the equation of the mirror line for a given object and its image.	

 Include examples where the sides of the shape are not parallel or perpendicular to the mirror line (vertical, horizontal and diagonal, including through the shape).		
7Gp.05 Rotate shapes 90º and 180º around a centre of rotation, recognising that the image is congruent to the object after a rotation.	**8Gp.05** Understand that the centre of rotation, direction of rotation and angle are needed to identify and perform rotations.	
Learners should understand that for 90º rotation, they will need to consider if the rotation is clockwise or anticlockwise.		
7Gp.06 Understand that the image is mathematically similar to the object after enlargement. Use positive integer scale factors to perform and identify enlargements.	**8Gp.06** Enlarge 2D shapes, from a centre of enlargement (outside or on the shape) with a positive integer scale factor. Identify an enlargement and scale factor.	**9Gp.06** Enlarge 2D shapes, from a centre of enlargement (outside, on or inside the shape) with a positive integer scale factor. Identify an enlargement, centre of enlargement and scale factor.
Ensure learners understand that enlargements preserve angles but not lengths. Sometimes the enlarged image can surround or overlap the original image.	Learners should recognise the invariant properties when carrying out an enlargement from a centre, and the effect on the lengths and ratios of sides.	e.g. describe the transformation that maps the object onto its image (enlargement, scale factor 3, centre of enlargement (1, 5)) Ensure learners understand what is needed to give a precise description of an enlargement.
		9Gp.07 Analyse and describe changes in perimeter and area of squares and rectangles when side lengths are enlarged by a positive integer scale factor.
		e.g. a square with sides 2cm has perimeter 8cm and area 4cm^2. If you double the sides, the perimeter will double (16cm) but the area will be times 4 (16cm^2).

		e.g. the area of a rectangle is $38cm^2$. The rectangle is enlarged by a scale factor of 3. What will be the area of the enlarged rectangle?

Statistics and Probability
Statistics

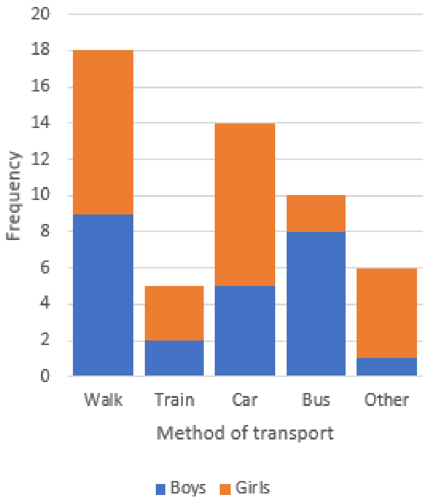
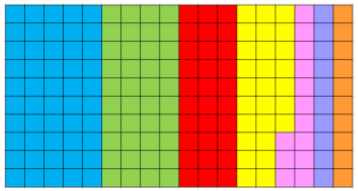
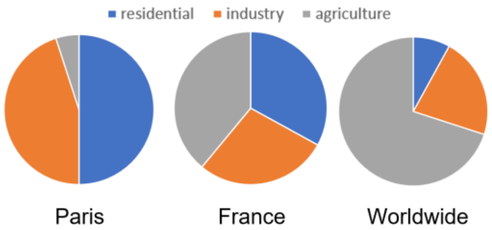
Stage 7	Stage 8	Stage 9
7Ss.01 Select and trial data collection and sampling methods to investigate predictions for a set of related statistical questions, considering what data to collect (categorical, discrete and continuous data).	**8Ss.01** Select, trial and justify data collection and sampling methods to investigate predictions for a set of related statistical questions, considering what data to collect (categorical, discrete and continuous data).	**9Ss.01** Select, trial and justify data collection and sampling methods to investigate predictions for a set of related statistical questions, considering what data to collect, and the appropriateness of each type (qualitative or quantitative; categorical, discrete or continuous).
Ensure learners can identify whether data is categorical, discrete or continuous data. Learners should select how they will collect their data. e.g. observation, questionnaire, interview, focus groups. Learners should select how they will choose their sample. e.g. drawing names out of a hat, using a random number generator, picking every 3rd person on the class register, choosing the first 50 people in the lunch queue.	Ensure learners understand that there are some data collection methods and sampling techniques that are more suitable for a specific type of data. Once a learner has selected their data collection and sampling methods they should be able to justify the reasons for their choice. e.g. "I put all the girls' names in one hat and all the boys' names in another. Then I pulled out 20 boys' names and 10 girls' names. This was because there are double the number of boys in my school than there are girls, so I wanted to ensure my sample was representative of the boys and girls in the population" Ensure learners consider alternative data collection and sampling techniques. e.g. "I could have given out a questionnaire instead of interviewing. This would have taken me less time, but I wouldn't be as sure that people had understood the questions".	Include examples where learners need to decide what data to collect in order to answer the question. Ensure learners consider the appropriateness of each data type. e.g. "For word length I could measure the length of each word with a ruler, or simply count the letters. Counting letters is more appropriate as it is simpler/ easier, and for some fonts the spaces between the letters is not consistent which creates an extra source of variation, plus it is hard to measure accurately which will also add a source of variation."
7Ss.02 Understand the effect of sample size on data collection and analysis.	**8Ss.02** Understand the advantages and disadvantages of different sampling methods.	**9Ss.02** Explain potential issues and sources of bias with data collection and sampling methods, identifying further questions to ask.

Ensure learners understand what a population and sample are. They should understand the importance of sample size prior to starting an investigation. e.g. if learners are collecting data for a whole country, the sample size should be bigger than if they are collecting data from the schools (in the latter, it could even be the whole population). Ensure learners understand that the larger the sample, the longer it may take to collect and analyse the data. However, if the sample is too small, it may not be representative of the population. As the sample grows the proportions in the sample reflect the population more closely, but there comes a point where increasing the sample size does not meaningfully improve the correspondence between sample/population proportions (the proportions "stabilise").	e.g. what sampling method could you use if you wanted to ask learners in your school about school lunch. What would be the advantages and disadvantages of each? • Selecting 5 learners from each tutor group in the school • Picking the first 50 from an alphabetical list of all learners who attend the school • Choosing 50 names out of a hat which contains the names of all learners who attend the school • Asking the first 50 people in the lunch queue.	e.g. if a population is made of 45% female and 55% male, and the sample has 10% male and 90% female. e.g. Standing outside a swimming pool asking people if they like swimming. e.g. Learners are using data from an outdated survey about social media that included some that are not used anymore and did not include current platforms. Learners should understand the sources of bias may be intentional. A biased sample may make the data appear more positive e.g. on advertising campaigns. They should identify further information required to determine whether the data was collected in a fair way.

Stage 7	Stage 8	Stage 9
7Ss.03 Record, organise and represent categorical, discrete and continuous data. Choose and explain which representation to use in a given situation: - Venn and Carroll diagrams - tally charts, frequency tables and two-way tables - dual and compound bar charts - waffle diagrams and pie charts - frequency diagrams for continuous data - line graphs - scatter graphs - infographics.	**8Ss.03** Record, organise and represent categorical, discrete and continuous data. Choose and explain which representation to use in a given situation: - Venn and Carroll diagrams - tally charts, frequency tables and two-way tables - dual and compound bar charts - pie charts - frequency diagrams for continuous data - line graphs and time series graphs - scatter graphs - stem-and-leaf diagrams - infographics.	**9Ss.03** Record, organise and represent categorical, discrete and continuous data. Choose and explain which representation to use in a given situation: - Venn and Carroll diagrams - tally charts, frequency tables and two-way tables - dual and compound bar charts - pie charts - line graphs, time series graphs and frequency polygons - scatter graphs - stem-and-leaf and back-to-back stem-and-leaf diagrams - infographics.

<table>
<tr>
<td>Lists, tables, tally</td>
<td>Representing data in two way tables:
<table>
<tr><th></th><th>Primary</th><th>Lower Secondary</th><th>Total</th></tr>
<tr><td>Walk</td><td>16</td><td>12</td><td>28</td></tr>
<tr><td>Bike</td><td>3</td><td>17</td><td>20</td></tr>
<tr><td>Car</td><td>18</td><td>5</td><td>23</td></tr>
<tr><td>Taxi</td><td>4</td><td>1</td><td>5</td></tr>
<tr><td>Bus</td><td>5</td><td>19</td><td>24</td></tr>
<tr><td>Total</td><td>46</td><td>54</td><td>100</td></tr>
</table>
</td>
<td>Tabulate discrete and continuous data, choosing suitable class intervals where appropriate. Use of ≤, < for continuous data intervals, e.g.
<table>
<tr><th>Mass (kg)</th><th>Frequency</th></tr>
<tr><td>$0 \le m < 50$</td><td></td></tr>
<tr><td>$50 \le m < 100$</td><td></td></tr>
<tr><td>$100 \le m < 150$</td><td></td></tr>
</table>
</td>
<td>Continue to use tally charts, frequency tables and two-way tables where appropriate and explain choice of data representation.
Use examples that require learners to regroup data in tables e.g. regroup intervals of 5 into intervals of 10 or vice versa.</td>
</tr>
<tr>
<td>Venn and Carroll</td>
<td>Use Venn and Carroll diagrams where appropriate and explain choice of data representation.</td>
<td>Continue to use Venn and Carroll diagrams where appropriate and explain choice of data representation.</td>
<td>Continue to use Venn and Carroll diagrams where appropriate and explain choice of data representation.</td>
</tr>
<tr>
<td>Block graph, pictogram, bar chart</td>
<td>Introduce dual and compound bar charts with two categories
e.g. dual bar chart:


</td>
<td>Continue to use dual and compound bar charts where appropriate and explain choice of data representation.
Include compound bar charts with more than two categories for each bar.
Ensure learners are confident drawing bar charts with more complicated scales.</td>
<td>Continue to use dual and compound bar charts where appropriate and explain choice of data representation.
Ensure learners can convert information between dual and compound bar charts.</td>
</tr>
</table>

	e.g. compound bar chart:  Include bar charts with grouped discrete data in any sized groups e.g. in 3s (1–3, 4–6, 7–9 etc.)		
Waffle diagrams and pie charts	Use examples where the total frequency is a factor of 360. Use waffle diagrams that use more than 100 squares so that learners calculate the percentage and proportion of each colour, then apply this to pie charts. e.g. 	Include pie charts that represent different totals to compare proportion, e.g.  Ensure learners can draw pie charts where the total frequency is not a factor of 360.	Ensure learners can convert information represented in bar charts to pie charts and vice versa.

<table>
<tr>
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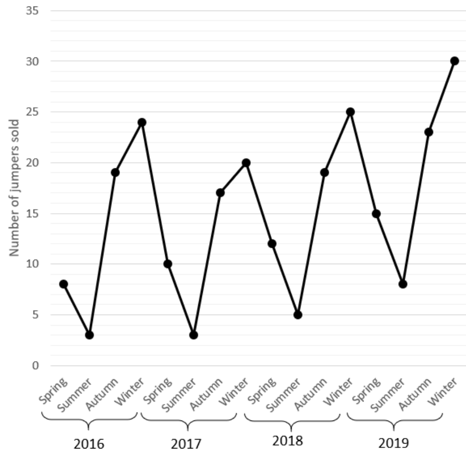
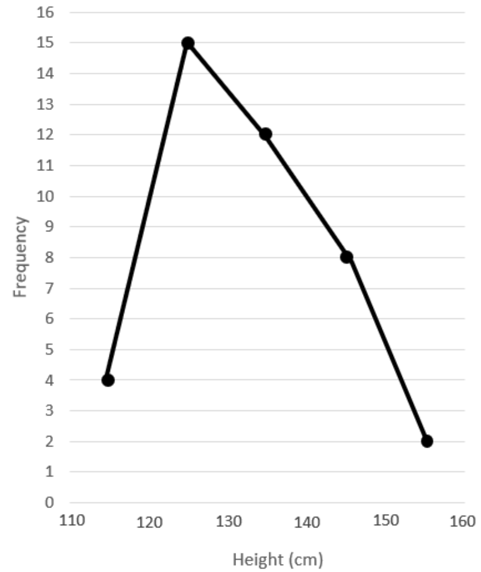
<tr>
<td>Frequency diagrams</td>
<td>Continue to use frequency diagrams for continuous data where appropriate and explain choice of data representation.

Learners should be confident in drawing frequency diagrams for continuous data from the beginning, including choosing their own scales for frequency.</td>
<td>Continue to represent data as frequency diagrams for continuous data where appropriate and explain choice of representation.

Ensure learners can draw frequency diagrams from tables which use ≤, < for continuous data intervals.</td>
<td></td>
</tr>
<tr>
<td>Line graphs</td>
<td>Continue to use line graphs where appropriate and explain choice of data representation.

Include curved lines that require interpretation</td>
<td>Introduce time series graphs to show a trend over time, e.g.


</td>
<td>Use frequency polygons to represent frequency of grouped data


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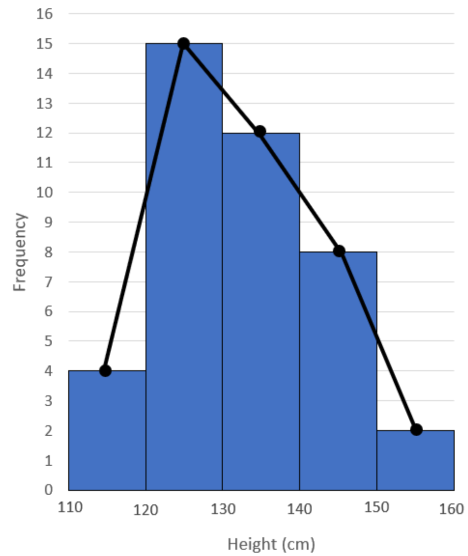
			Ensure learners see the relationship between frequency polygons and frequency diagrams for continuous data, e.g. 
Scatter graphs	Continue to use scatter graphs where appropriate and explain choice of data representation. Use scales such as 1s, 2s, 5s, 10s or decimals such as 0.5s or 0.1s.	Continue to use scatter graphs where appropriate and explain choice of data representation. Ensure learners can draw scatter graphs to compare data. For example, plot boys' and girls' data on the same graph using a key, or on different graphs, but using the same scales.	Use examples that develop understanding of correlation (positive, negative and zero correlation; draw the line of best fit and use it as a model for the relationship). Ensure learners can also describe the strength of the correlation as strong or weak.
Infographics	Represent data as an infographic.	Continue to use infographics where appropriate and explain choice of data representation.	Continue to use infographics where appropriate and explain choice of data representation.
Stem and leaf		Introduce presenting data in stem-and-leaf diagrams	Introduce presenting data in back-to-back stem-and-leaf diagrams

Stage 7

7Ss.04 Use knowledge of mode, median, mean and range to describe and summarise large data sets. Choose and explain which one is the most appropriate for the context.

Learners should be able to calculate summary statistics for data presented in a frequency table.

At this stage they do not need to be able to calculate the mean from a grouped frequency table.

e.g. This is the summary of how many pairs of shoes of each size a shop sold last month.

Shoe size	Frequency
4	4
5	9
6	14
7	11
8	2

Find the mean shoe size, the median shoe size, the modal shoe size, and the range of the shoe sizes.

Ensure learners understand what each of these measures mean for this context.

Understand that the median of a data set represents the central value when the data is placed in order. The median tends not to be affected by extreme values. The mean calculation takes into account all data items and so can be effected by extreme values. If someone with a shoe size of 14 was added to the above data set, consider how it would affect the value of the mean, median, mode and range.

Stage 8

8Ss.04 Use knowledge of mode, median, mean and range to compare two distributions, considering the interrelationship between centrality and spread.

Learners should be able to compare summary data for two or more distributions.

e.g. Spelling test results (out of 10)

	Boys	Girls
Mean	7.1	8.3
Mode	9	8
Median	6	8
Range	9	5

The mean and median suggests that the girls did better in the spelling test.

The range suggests that the girls scores were closer together and so more consistent.

Why might the median be a better summary statistic to use for this data?

Use examples of:

- mode or modal class (include cases where there is no mode)
- median (not from grouped data)
- mean (not from grouped data)
- range (not from grouped data).

Stage 9

9Ss.04 Use mode, median, mean and range to compare two distributions, including grouped data.

Learners should be able to calculate an estimate for the mean, the class interval where the median lies, the modal class interval and an estimate for the range for grouped data.

e.g.

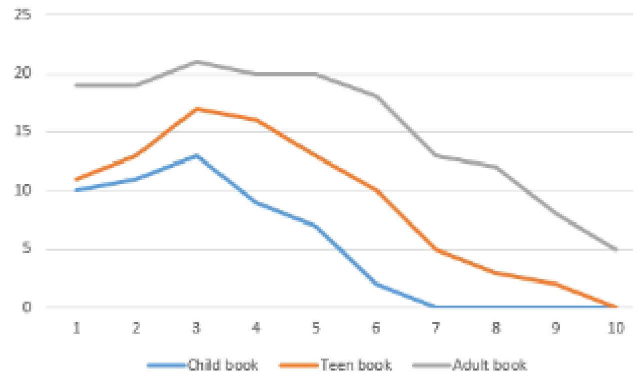
Test score	Frequency
$0 < t \leq 20$	4
$20 < t \leq 40$	11
$40 < t \leq 60$	15
$60 < t \leq 80$	12
$80 < t \leq 100$	8

Find an estimate for the mean, the interval where the median test score lies, the modal class interval, and an estimate for the range of the test scores.

Ensure learners understand that when calculating spread and centrality of grouped data:

- The mean is an estimate
- A precise range cannot be found
- The modal class changes depending on how the data is grouped (the mode may not lie in the modal class).

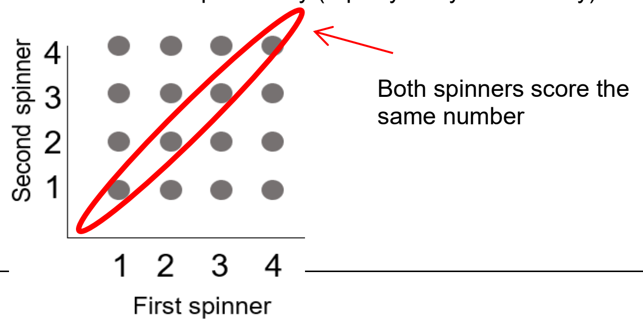
The ‘estimate for the range’ should be the maximum possible value of the range.

7Ss.05	8Ss.05	9Ss.05
7Ss.05 Interpret data, identifying patterns, within and between data sets, to answer statistical questions. Discuss conclusions, considering the sources of variation, including sampling, and check predictions.	**8Ss.05** Interpret data, identifying patterns, trends and relationships, within and between data sets, to answer statistical questions. Discuss conclusions, considering the sources of variation, including sampling, and check predictions.	**9Ss.05** Interpret data, identifying patterns, trends and relationships, within and between data sets, to answer statistical questions. Make informal inferences and generalisations, identifying wrong or misleading information.
Learners should be able to make informal inferences from sample to population. e.g. when looking at the number of 3-letter words for one page, be able to make a prediction for the whole book. Learners consider the sources of variation, including data sampling, and explain <u>why</u> the data shows variations. e.g. "We only collected data from one page. This page may or may not be representative of the rest of the pages in the book"	Learners identify <u>trends and relationships</u>.  e.g. looking at the frequency graphs for the length of all words from one page of a children's book, teenage book and adult book, learners may notice: • the shape of all three lines are similar but the teenage line is above the children's line and the adult line is above both • all three graphs peak at 3-letter words and go down at the end Learners should also relate their findings back to their predictions. They consider why the distributions are as they are and make suggestions for further investigations. e.g. "I thought the children's book would have the most 1-letter, 2-letter and 3-letter words, but the adult book has the most. This might be because the older the intended audience, the more words there are per page."	Ensure learners can analyse graphical data from the media and identify misleading statements represented, analyse and identify the elements that can induce errors sometimes on purpose (e.g. inappropriate scales, small sample size, omission of dates, etc.). When interpreting scatter graphs ensure learners understand that correlation between variables does not automatically mean that the change in one variable is the cause of the change in the values of the other variable. e.g. it may appear from a scatter graph showing age of adult (18+) and children's books bought in the past year, that the older you get the more you enjoy reading children's books. But the likelihood is these adults are buying the children's books for their children, not themselves. Learners should make inferences and generalisations. e.g. after analysing data for 3 children's books, 3 teenage books and 3 adults book they are able to say ""A book aimed at teenagers will most likely have between 200 and 250 pages and 300 words per page, whereas an adult book will have 600 pages and 400 words per page." Learners may wish to investigate extrapolation and how it may show misleading information.

	Learners should recommend how to refine the process for future investigations. e.g. “Next I could consider counting the length of the first 100 words of all three books instead of just the words on one page.”	

Statistics and Probability
Probability

Stage 7	Stage 8	Stage 9
7Sp.01 Use the language associated with probability and proportion to describe, compare, order and interpret the likelihood of outcomes.		
Use language such as fair, unfair, likely, unlikely, equally likely, even chance, certain, uncertain, probable, possible, impossible, likelihood, probability, random, outcome, bias, fair. Use examples where probability is shown as a fraction, decimal or percentage, not as a ratio or in words i.e. not ‘1:4’ and not ‘1 in 4’.		
7Sp.02 Understand and explain that probabilities range from 0 to 1, and can be represented as proper fractions, decimals and percentages.	**8Sp.01** Understand that complementary events are two events that have a total probability of 1.	**9Sp.01** Understand that the probability of multiple mutually exclusive events can be found by summation and all mutually exclusive events have a total probability of 1.
Ensure learners understand that probabilities range from 0 (impossible) to 1 (certain) so probabilities cannot be less than zero or higher than 1. The probability of flipping a coin and it landing on tails could be expressed as $\frac{1}{2}$, 0.5, or 50%, $\frac{1}{2}$ means one out of two or 1 outcome out of two possible outcomes. e.g. the probability of picking a diamond card from a pack of 52 playing cards is $\frac{13}{52}$ which simplifies to $\frac{1}{4}$ or 0.25 or 25%.	The complement of any event A is the event [not A] and can be represented as P (A) + P (A’) = 1 If the probability of an event occurring is *p*, then the probability of it not occurring is 1 – *p*. e.g. if the probability of landing on a red section on a spinner is 0.3 then the probability of landing in any other colour (not landing on a red section) is 0.7	e.g. There are 12 cars. 5 are red, 2 are blue, 1 is white and the rest are black. A car is chosen at random. Find the probability that a car is black. e.g. A bag contains some cards with numbers 1 to 4 on. The probabilities of choosing at random a card marked with a particular number are: P(1) = 0.15 P(2) = 0.1 P(3) = 0.35 Find the probability of choosing a card with a 3 or 4 on it.
7Sp.03 Identify all the possible mutually exclusive outcomes of a single event, and recognise when they are equally likely to happen.	**8Sp.02** Understand that tables, diagrams and lists can be used to identify all mutually exclusive outcomes of combined events (independent events only).	**9Sp.02** Identify when successive and combined events are independent and when they are not.

Ensure learners understand the term mutually exclusive. A sports team can win, lose or draw but these events cannot happen at the same time. Emphasise that some probabilities cannot be calculated by using equally likely outcomes but can be modelled through experiments involving a large number of trials. e.g. the probability of a drawing pin landing point up; rolling a fair die; tossing a coin.	E.g. How many different three-digit numbers can be made using the digits 3, 6, and 9? First, find all the numbers that can start with 3: 369 and 396 There will also be two numbers that start with 6 and 9: 639 and 693 936 and 963 So there are six possible numbers: 369 396 639 693 936 963 Ensure learners recognise different methodical listing strategies for combined events such as tables, tree diagrams and ordered lists.	Ensure learners understand the term independent. Recognise how successive and combined events can affect each other. e.g. There are 3 blue discs and 5 red disks in a bag. One disc is taken at random and not replaced. Then another is taken. e.g. Event A is the probability of being late for school. Event B is the probability of it snowing. Does either of these events affect the other?
7Sp.04 Understand how to find the theoretical probabilities of equally likely outcomes.	**8Sp.03** Understand how to find the theoretical probabilities of equally likely combined events.	**9Sp.03** Understand how to find the theoretical probabilities of combined events.
Emphasise understanding of mutually exclusive events rather than using the language. Ensure learners understand that when outcomes are equally likely, the theoretical probability of any outcome happening is $\frac{1}{n}$ where n equals the number of outcomes. e.g. the probability of rolling a 3 on a fair die is $\frac{1}{6}$ The probability of rolling a number less than 6 is $\frac{5}{6}$	Find and list systematically all possible mutually exclusive outcomes for two successive events including: • use of possibility diagrams, • sample space diagrams, • Venn diagrams and • tree diagrams. Then calculate the probability (equally likely events only). 	Calculate the probability of simple combined events (including with outcomes which are not equally likely), with or without using tree diagrams (including with outcomes which are not equally likely, no conditional probability). Tree diagrams with probabilities on branches

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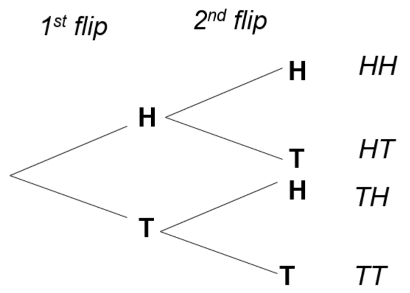
<tr><td></td><td colspan="6">Red dice</td></tr>
<tr><td>+</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>Green dice 1</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>2</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>3</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>4</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>5</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>6</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>
Tree diagrams can be introduced for recording outcomes but not for finding probabilities by multiplying along branches.
e.g. the probability of getting one head and one tail when throwing 2 coins ($\frac{2}{4}$)


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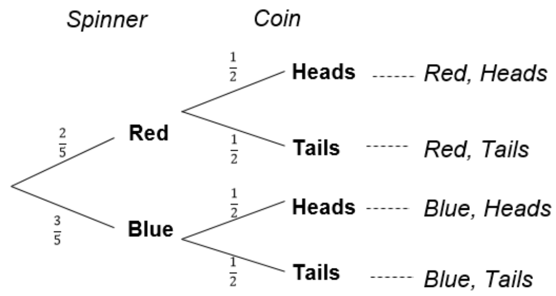

Ensure learners understand that the sums of joined branches add to 1.
</td>
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<tr>
<td>7Sp.05 Design and conduct chance experiments or simulations, using small and large numbers of trials. Analyse the frequency of outcomes to calculate experimental probabilities.</td>
<td>8Sp.04 Design and conduct chance experiments or simulations, using small and large numbers of trials. Compare the experimental probabilities with theoretical outcomes.</td>
<td>9Sp.04 Design and conduct chance experiments or simulations, using small and large numbers of trials. Calculate the expected frequency of occurrences and compare with observed outcomes.</td>
</tr>
<tr>
<td>Ensure learners understand that:
• when experiments are repeated different outcomes may result</td>
<td>e.g. Compare the estimated experimental probabilities of rolling different numbers on a 6 sided dice with the theoretical probabilities. Use this comparison to say whether or not the dice is fair.</td>
<td>e.g. Roll two dice and add the numbers shown to get a score. Repeat this 50 times. Use the data to estimate the probability of each score. Repeat another 50 times and compare the two sets of results, what do you notice?</td>
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</table>

• increasing the number of times an experiment is repeated generally leads closer to the theoretical outcomes • relative frequency is an estimate of probability Understand that randomness has uncertain individual outcomes but exhibit regular patterns of outcomes over many repetitions. e.g. drop a drawing pin and record if it lands point up or point down. Record how the experimental probability changes as the number of trials increases.		Draw a graph to show how the relative frequency of an outcome changes as the number of trials increases.

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