

Table 1: Classification of the GHD family ( $\alpha$  and  $\beta$  are interchangeable)

	Parameters	Conditions	Range	Type
$\gamma > 0$	$\alpha, \beta > 0$	$\gamma > \alpha + \beta$ if $\lambda = 1$	$[0, \infty)$	I
	$0 < \lambda \leq 1$ $\alpha, \beta \in \mathbb{C},$ $\alpha = \bar{\beta}$	$\gamma > \alpha + \beta$ if $\lambda = 1$	$[0, \infty)$	II
	$\alpha, \beta < 0,$ $\alpha, \beta \notin \mathbb{Z}^-$	$[\alpha] = [\beta]$	$[0, \infty)$	III
	$0 < \lambda$ $\alpha, \beta < 0,$ $\alpha \in \mathbb{Z}^-$	$ \beta  >  \alpha  - 1$	$[0,  \alpha )$	IV
	$\lambda < 0$ $\alpha \in \mathbb{Z}^-, \beta > 0$		$[0,  \alpha )$	V
$\gamma < 0$	$0 < \lambda \leq 1$ $\alpha < 0, \alpha \notin \mathbb{Z}^-,$ $\beta > 0$	$[\alpha] = [\gamma],$ $\gamma > \alpha + \beta$ if $\lambda = 1$	$[0, \infty)$	VI
	$0 < \lambda$ $\alpha \in \mathbb{Z}^-, \beta > 0$	$ \gamma  >  \alpha  - 1$	$[0,  \alpha )$	VII
	$\lambda < 0$ $\alpha, \beta < 0,$ $\alpha \in \mathbb{Z}^-$	$ \gamma ,  \beta  >  \alpha  - 1$	$[0,  \alpha )$	VIII